Department of Physics, Chemistry and Biology

Bachelor's Thesis

Low cost DC-linearity testing of long scale Digital Voltmeters

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Sammanfattning
Abstract

The presented thesis focuses on a low cost method to measure the linearity of Digital Voltmeters. The developed method would cost several orders of magnitude less for implementation and operation than conventional methods. For state of the art calibration level multimeters only typical values for linearity are given, since linearity is only spot-tested on selected units during manufacturing.

The suggested method requires three or more stable resistors and a stable voltage source. From these components a voltage divider was built. The voltages over several series combinations of the resistors were measured. The Kirchhoff's voltage law was applied to the circuit in order to produce a system of equations. From this system of equations a limit for the non-linearity was calculated. The suggested method is tested using both, artificially generated data as well as measured data from a number of multimeters.

All estimations for the non-linearities were found to be consistent with the theoretical limits of the multimeters and the noise levels of the voltage sources.

Nyckelord Keywords

Linearity, Long scale DVM, Digital Voltmeter

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The presented thesis focuses on a low cost method to measure the linearity of Digital Voltmeters. The developed method would cost several orders of magnitude less for implementation and operation than conventional methods. For state of the art calibration level multimeters only typical values for linearity are given, since linearity is only spot-tested on selected units during manufacturing.

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Contents

Chapter 1

Introduction

1.1 Motivation

Today's state of the art multimeters have very good performance when it comes to linearity. Even though these multimeters cost a premium, the cost and time required to test their linearity is even higher. The most expensive multimeters today cost about 100000 USD, the equipment required to test its linearity will cost 10 times that. Due to the amount of time required to perform the testing this will never be performed as part of production testing.

The process of calibration and verification of measurement equipment against known traceable references is of very high importance for the reliability and traceability of measurements. Unfortunately this verification will only verify an instrument at certain fixed values.

Usually it is possible to use secondary equipment to divide the reference value to a fixed lower value, 1/2 or 1/10 for example, but this will always introduce additional uncertainties due to the required calibration of the divider unit.

In the case of Digital Volt Meters (DVM) the traditional method used a Kelvin-Varley-Divider (KVD) to set very precise division ratios, down to 0.1 ppm. To reach this amount of precision the KVD needs to be fully calibrated every few hours, a process that requires a fair amount of experience and patience.

Recently the use of Josephson Junction Arrays[1] (JJA) has been used to very accurately, 0.01 ppm, set very fine voltage intervals. The drawback is that it is a very expensive and cumbersome piece of equipment. The JJA used by the Swedish institute Statens Provningsanstalt is only started twice per year for example.

Both of the above methods rely on the absolute accuracy of either the divider ratio or the exact voltage generated.

This thesis describes a method where only stability and the additive property is required to estimate an upper bound of the non-linearity. This suggested method is then tested using both artificial data as well as a number of different multimeters.

1.2 The Kelvin-Varley divider

Since 1866 the Kelvin-Varley divider (KVD) has been used to set voltage ratios with a very high precision. The Weston cell[2] voltage standard used to implement the volt up until 1990 was only accurate to at best 1 ppm so the 0.1 ppm possible with a KVD was good enough [3].

While a simple voltage divider draws some current from its source it must not be loaded by any output current or the output voltage will change from the ideal. In general it is therefore not possible to cascade several dividers after each other. The KVD solves this problem by having each successive divider string at a well determined resistance. The least significant string in figure 1.1 will have a total resistance of 10 R, this string will be in parallel with 2 of the resistors in the adjacent string so if those two have a total series resistance of 10 R the resistance over the entire middle string will be

$$9 \times 5 + (2 \times 5) \parallel (10 \times 1) = 50 \qquad [\mathrm{R}].$$

As long as no current is drawn from the output the resistive loading on each divider string in the KVD remains the same. This cascading can be continued indefinitely.

To calibrate a KVD it is only necessary to compare and adjust resistors to each other so that all resistors in a string have the same resistance. The resistors in the strings in figure 1.1 are compared to the other resistors in the same string using a Wheatstone-bridge and trimmed to the same value. Then the entire strings from right to left are trimmed, using a parallel trim-resistor, to have the same resistance as two resistors in the preceding string. For a large KVD the two or three lower value strings usually do not need trimming since the requirement is only 10^{-3}. For a 7-decade KVD this still means that 48 individual resistances has to be adjusted to a precision of down to 10^{-7}. The resistors will drift out of adjustment within hours so re-trimming will be required more than once per day [4].

1.3 The Josephson Junction Array

In 1988 the 18th General Conference on Weights and Measures accepted the Josephson Effect to define the volt.

The Josephson effect, where a current will flow through an insulator from one superconductor to another, was predicted in 1962 by Brian David Josephson [5].

If a DC voltage is applied to this junction the current will oscillate at a specific frequency while emitting or absorbing photons at this frequency. The frequency will only depend on the applied voltage, the electron charge and the Planck constant [6].

The reverse also happens, if a Josephson junction is subjected to EM-radiation it will develop a voltage only depending on frequency according to

$$U = \frac{\hbar}{2e}\omega\left(n + a\cos(\omega t)\right)$$

where the DC-part only depends on Planks constant $\hbar$, the electron charge e, an arbitrary integer n and the frequency ω. The value of a will be of low significance

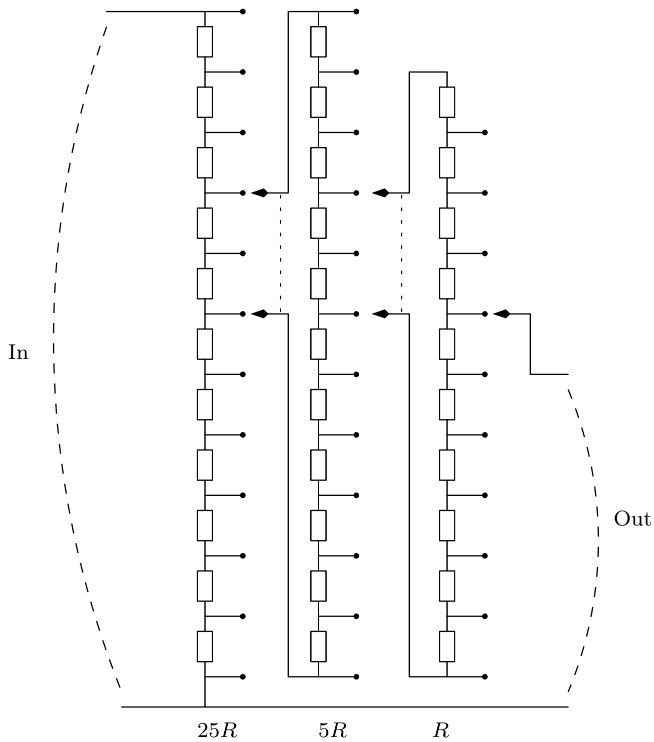

Figure 1.1. A 3-stage Kelvin-Varley divider.

here because that part of the expression will average to zero, it is related to the AC-current that is induced in the junction and will be related to the load placed on the junction [5, 6].

In practical applications the voltage over the junction is limited to some 100's of μV so a large number of junctions are connected in series [7] called a Josephson Junction Array (JJA).

A JJA has to be kept at low temperature, even though high temperature superconductors can be used [8, 9] there will still be a need for liquid LN2 cooling. They can also drop out of synchronization [10] with the induced microwave radiation and need restarting by a trained operator. More recent arrays are made using niobium and lead-indium-gold alloy superconductors and niobium oxide insulators.

Current research focus on Superconductor-metal-superconductor junctions[11, 12]. Recently a high temperature 10 V system has been developed using a combination of a low voltage JJA and a high performance Zener reference[13].

1.4 Summary of traditional methods

Both of the above mentioned methods require substantial investments, from 1000's to 10000's of Euros, and substantial set-up procedures and operator training.

1.5 Objectives

The purpose of this work is develop the theoretical framework and to experimentally test a new method for testing linearity of digital voltmeters. The method should be feasible both for the production-line user as well as the DIY-home user, and should also be simpler to perform than the common methods used for evaluating linearity today. It should require only a stable noise-free voltage source and stable noise-free resistors and should not require components with high absolute accuracy. The work will be divided into two main sections. In the first the theoretical base for the method is established and tested on artificial data sets. In the second part the method is tested and verified on a variety of long scale multimeters. The results will be compared to datasheet values and to the theoretical limits of linearity.

Chapter 2

Theory

In this chapter the theoretical base for the suggested method is presented. There after 2 examples are given as well as 3 tests using artificially constructed data performed.

2.1 Notation

- $g(x)$ is a scalar function, g is specifically the numerical value display when measuring v volts.
- $\mathbf{g}$ is a vector in the form of a column matrix. $\mathbf{g}$ is specifically the numerical values displayed by the DVM for each of the applied voltages in $\mathbf{v}$.
- $\mathbf{g} + b$ implies that b is added to each element of $\mathbf{g}$, an addition of b with the appropriate column vector of all ones.
- $\mathbf{e}$ is the error-, or non-linearity-vector.
- $\mathbb{F}$ is a system of equations. $\mathbb{F}$ is specifically the consistency requirements for voltages in a resistive divider.
- $\mathbb{F}^+$ is the Moore-Penrose pseudo inverse of $\mathbb{F}$. For an under-specified system of equations, $\mathbb{F}^+$ will give the solution with the lowest possible euclidean length.
- $\mathbf{g}(\mathbf{x})$ is a vector of g applied element-wise to $\mathbf{x}$.

2.2 Definition of linearity

An operator L is linear iff:

$$L(x + y) = L(x) + L(y) \tag{2.1}$$
$$L(ax) = aL(x) \tag{2.2}$$

(2.1) is the additive criteria and (2.2) the homogeneity criteria, (2.2) is a consequence of (2.1) if L is continuous and only (2.1) will be used here.

2.2.1 Affine operators

An affine operator will preserve straight lines, as a linear operator, but might not preserve the zero-point:

$$A(x + y - b) = A(x - b) + A(y - b) \tag{2.3}$$

where b can be interpreted as a translation. Non-linearity is usually defined as the deviation from an affine operator.

2.2.2 Measures of non-linearity

Several different measurements to evaluate non-linearity exist: Independent Linearity, Zero-based Linearity, Terminal Linearity and Absolute Linearity.

All of these measurements measure the deviation of the system from a straight line, the difference lies in how that straight line is fitted to the true response.

- Independent Linearity measures the deviation from a straight line where the line is oriented so to minimize the deviation. See figure 2.1.
- Zero-based Linearity constrains one end of the line (the Zero-end). See figure 2.2.
- Terminal Linearity constrains both endpoints of the line to the actual endpoints as measured. See figure 2.3.
- Absolute Linearity constrains the line to the true values of what is measured, in this case both offset and gain errors will be included in the non-linearity. See figure 2.4.

The amount of non-linearity can be quantified using several different units. Throughout this report ppm of range will be used exclusively. Other common units are ppm of measured value, measured quantity (μV) or digits (bits).

In the field of measurements, "linear" are usually interpreted the same way as affine in mathematics, in applied electronics "linear" can even be translated to continuous in mathematics but if the topic is non-linearity it means "affine operator".

2.2.3 Almost linear (affine) function

An affine function can be written:

$$y = ax + b \tag{2.4}$$

Almost affine or almost linear (as in follows a straight line) function:

$$y = ax + b + e(x) \tag{2.5}$$

where $e(x)$ is small.

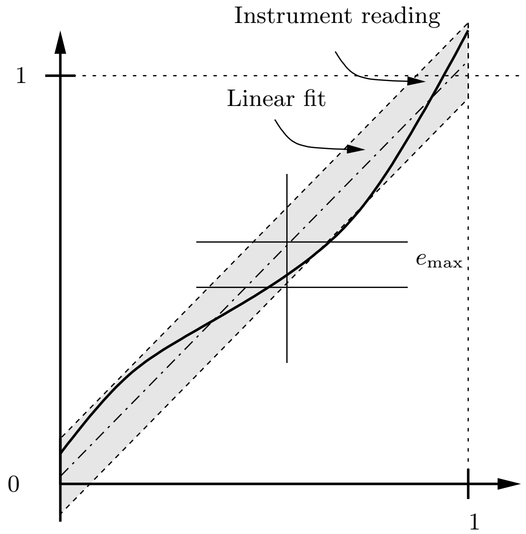

Figure 2.1. Illustration of independent linearity. The linear fit *does not* have to intercept the y-axis at zero and can have any slope to minimize $e_{\max}$. Measurement data will be within the gray area.

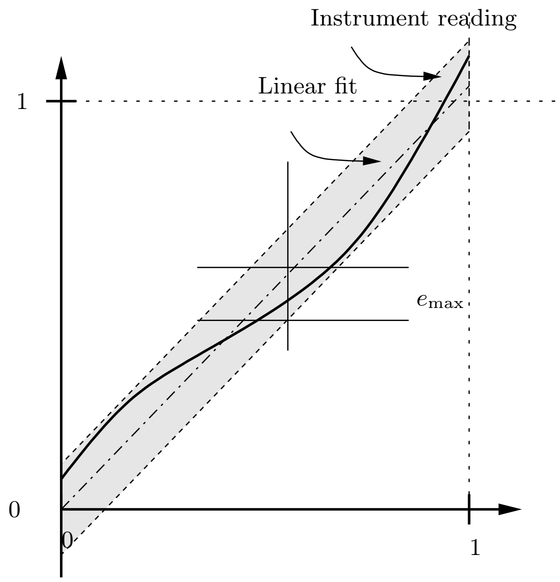

Figure 2.2. Illustration of zero-based linearity. The linear fit *does* have to intercept the y-axis at zero and can have any slope to minimize e. Measurement data will be within the gray area.

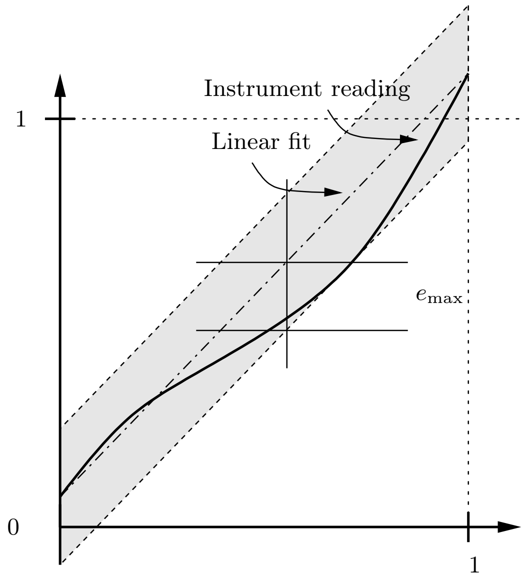

Figure 2.3. Illustration of terminal linearity. The linear fit has to intercept the y-axis at the instrument reading and has to intercept the instrument reading at the terminal value. Measurement data will be within the gray area.

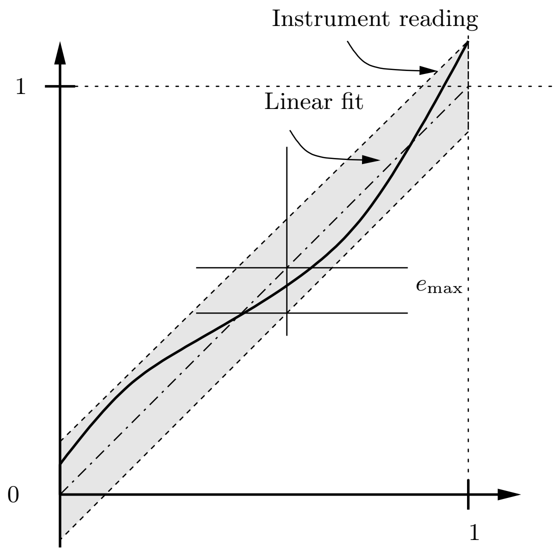

Figure 2.4. Illustration of absolute linearity. The linear fit *does* have to intercept the y-axis at zero and have the true value at the terminal value, i.e. there is no freedom in the linear approximation and all errors are interpreted as non-linearity deviations. Measurement data will be within the gray area.

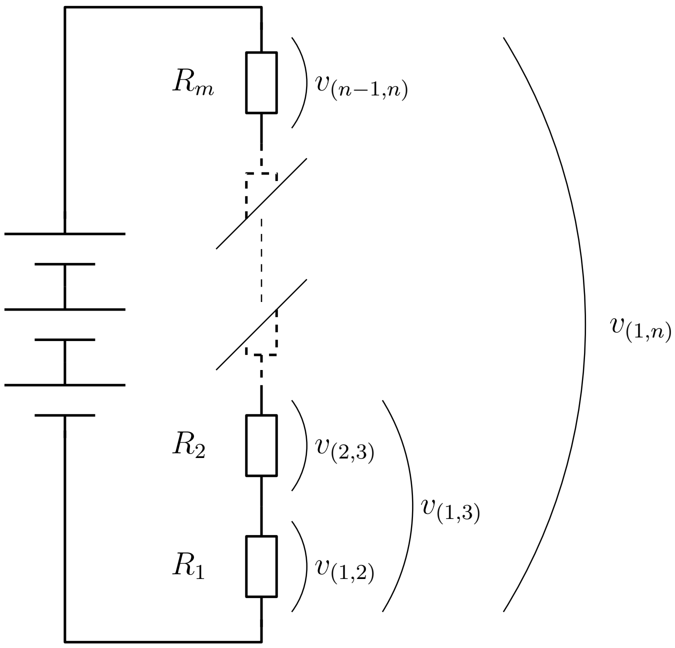

Figure 2.5. A resistive divider with a voltage supply.

2.2.4 Estimating the deviation from a straight line

A given real-world system $g(v)$ that is almost linear can be characterized over some points in a certain range.

$$\mathbf{g}(\mathbf{v}) = a\mathbf{v} + b + \mathbf{e}(\mathbf{v}). \tag{2.6}$$

In a general resistive divider we know (from Kirchhoff's voltage law) that the voltages over series-combinations of the resistors will add up as:

$$v_{(1,3)} = v_{(1,2)} + v_{(2,3)} \tag{2.7}$$

with variables defined as in figure 2.5. The divider does not have to be resistive, nowhere Ohms law is used,

From this we can formulate a system of equations that should hold for a perfectly linear system:

$$\begin{array}{rcl} g(v_{(1,2)}) + g(v_{(2,3)}) & = & g(v_{(1,3)}) \\ g(v_{(2,3)}) + g(v_{(3,4)}) & = & g(v_{(2,4)}) \\ & \vdots & \\ g(v_{(1,2)}) + g(v_{(2,n)}) & = & g(v_{(1,n)}) \end{array} \tag{2.8}$$

from here on this system of equations will be labeled $\mathbb{F}$ so that the above equation can be written

$$\mathbb{F}\mathbf{g} = \mathbf{0}.$$

One of the properties of $\mathbb{F}$ is that if it is applied to a column vector with all elements the same number the result will be a vector of different size but will all elements that same number, if $\mathbf{b}$ is a column vector with all elements b then each row in $\mathbb{F}\mathbf{b}$ will be $b + b - b = b$.

We do know from 2.7 that $\mathbb{F}\mathbf{v} = \mathbf{0}$. For a real-world system $\mathbb{F}\mathbf{g} = \mathbf{0}$ will not hold but by inserting 2.6 into 2.8 we get:

$$\begin{aligned} & g_{(1,2)} + g_{(2,3)} - g_{(1,3)} \\ & \quad = av_{(1,2)} + b + e_{(1,2)} + av_{(2,3)} + b + e_{(2,3)} - av_{(1,3)} - b - e_{(1,3)} \\ & \quad = b + e_{(1,2)} + e_{(2,3)} - e_{(1,3)} \end{aligned} \tag{2.9}$$

or alternatively

$$g_i = av_i + e_i + b \tag{2.10}$$

and in matrix form:

$$\mathbb{F}\mathbf{g} = \underbrace{a\mathbb{F}\mathbf{v}}_{=0} + \mathbb{F}\left(\mathbf{e} + b\right) \tag{2.11}$$

If the order of the variables and equations are done according to the format described in section A and the matrix $\mathbb{F}$ defined from the program the full system can be written:

$$\mathbb{F}\left(\mathbf{e} + b\right) = \mathbb{F}\mathbf{g} \tag{2.12}$$

This system will have $\frac{n(n-1)}{2}$ measurements, g, and errors, e, but only $\frac{(n-1)(n-2)}{2}$ equations, rows in $\mathbb{F}$. Obviously this problem does not have a unique solution. As formalized here the behaviour of the system can be characterized using 3 parameters, gain-error, offset-error and non-linearity. The gain- and offset-errors must be chosen to minimize the non-linearity, the deviation from a straight line.

2.2.5 Solving the equations

The set of measurement points and error-estimates will always form an under-determined system, but since the non-linearity will be the minimum of the possible solutions one simply picks the smallest solution. This will be the Least Absolute Deviation (LAD or Minimum L^∞-solution). As the solution with the shortest square length, minimum euclidean norm, (Least Square Deviation, LSD), is easier to calculate, commonly implemented in software libraries and also guaranteed to have larger maximum deviation than the LAD-solution the LSD-solution is an option. In this thesis the `pinv` subroutine in the numPy software library, part of the Python universe, has been used to calculate the inverse. The solving of this under-determined system is equivalent to fitting a straight line to a set of data-points. The LSD-solution will also at worst show twice the maximum deviation compared to the LAD-solution, this is illustrated in figure 2.6.

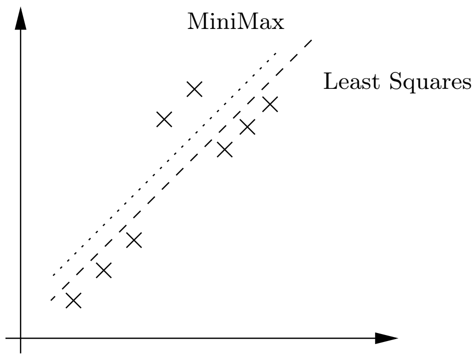

Figure 2.6. Two fittings to a number of data points. The dotted line has the smallest possible maximum deviation to any data point. The dashed line is approximately where a Minimum Square fitting would be.

Using the Moore-Penrose pseudo-inverse the system can be solved to minimize the euclidian norm of $\mathbf{e}$ [14].

$$\mathbf{e} = \mathbb{F}^{+}(\mathbb{F}\,(\mathbf{g} - b)) \tag{2.13}$$

The offset error can be estimated by the average residual of all the equations as:

$$b = \overline{\mathbb{F}\mathbf{g}}$$

.

2.3 Examples

2.3.1 Example 1

For illustration a voltmeter and divider with an assumed characteristic is used. This DVM will show the correct value of 0.00 V with zero input, correct value at 1.00 V and the wrong value of 0.40 V when the input is 0.50 V, see figure 2.8. Between these points the DVM is perfectly linear. The divider has two resistors of exactly the same value and a ratio of 1/2, see figure 2.7.

The absolute and terminal non-linearity is 0.1 since the measured value is 0.1 below the actual value. The independent non-linearity is half of that, 0.05, since this will be measured against the straight line with the lowest possible deviation from the measured values.

$$\mathbf{g} = \begin{pmatrix} 0.4 \\ 0.4 \\ 1.0 \end{pmatrix} \tag{2.14}$$

$$\mathbb{F} = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \tag{2.15}$$

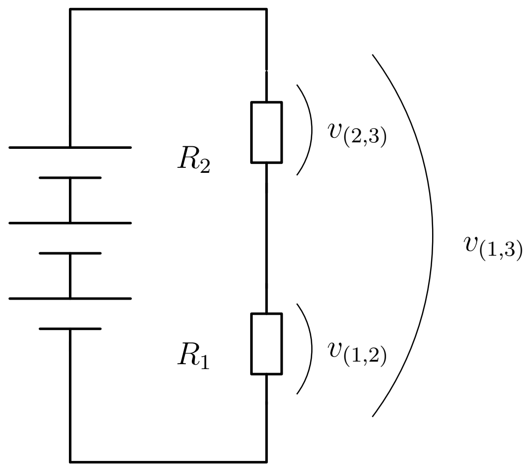

Figure 2.7. Illustration of the resistor network used in example 1.

$$\mathbb{F}^{+} = \begin{pmatrix} 1/3 \\ 1/3 \\ -1/3 \end{pmatrix} \tag{2.16}$$

$$b = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0.4 \\ 0.4 \\ 1.0 \end{pmatrix} = -0.2 \tag{2.17}$$

with numbers:

$$\begin{pmatrix} 1/3 \\ 1/3 \\ -1/3 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0.4+0.2 \\ 0.4+0.2 \\ 1.0+0.2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

In this case, any non-linearity can be compensated by an offset as far as the calculations go. The calculations become meaningless. This will happen for every case where there is only one equation:

$$g_{(1,2)} + g_{(2,3)} - g_{(1,3)} = e_{(1,2)} + e_{(2,3)} - e_{(1,3)} - b$$

so next we try with three resistors, 0.1 R, 0.3 R and 0.6 R, see figure 2.9.

2.3.2 Example 2

The DVM in this case will read low by 0.05 on the lowest voltage, see figure 2.10.

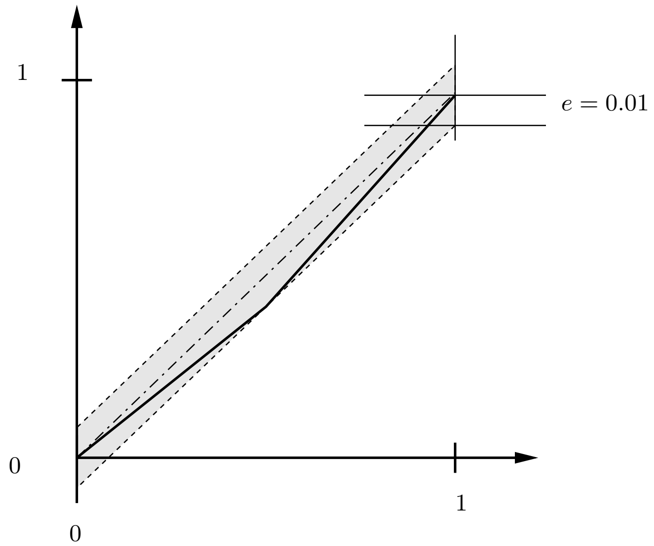

Figure 2.8. Illustration of the behavior of the DVM in example 1.

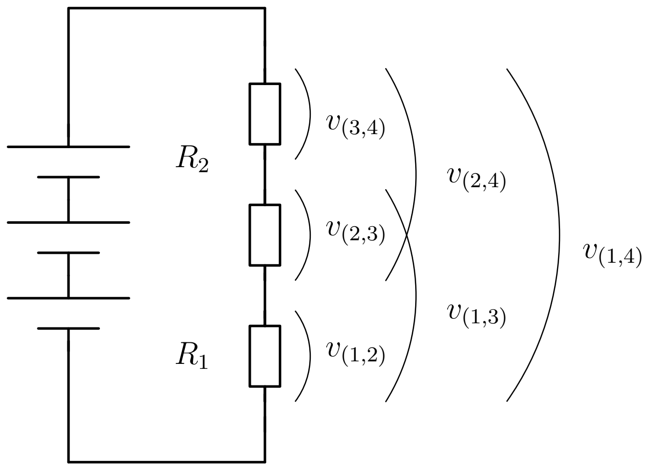

Figure 2.9. Illustration of the resistor network used in example 2.

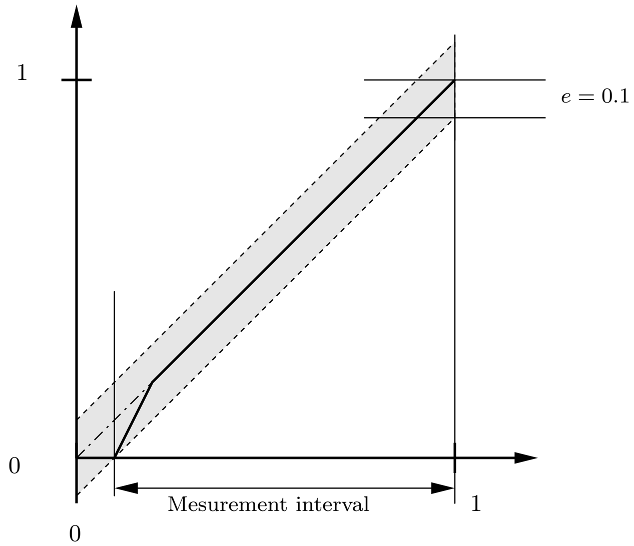

Figure 2.10. Illustration of the behavior of the DVM in example 2.

$$\mathbf{g} = \begin{pmatrix} 0.0 \\ 0.3 \\ 0.6 \\ 0.4 \\ 0.9 \\ 1.0 \end{pmatrix} \tag{2.18}$$

$$\mathbb{F} = \begin{pmatrix} 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \tag{2.19}$$

$$\mathbb{F}^{+} = \begin{pmatrix} 1/4 & 0 & 1/4 \\ 1/4 & 1/4 & 0 \\ -1/4 & 1/2 & 1/4 \\ -1/2 & 1/4 & 1/4 \\ 0 & -1/4 & 1/4 \\ 1/4 & -1/4 & -1/2 \end{pmatrix} \tag{2.20}$$

$$b = \overline{\begin{pmatrix} 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0.0 \\ 0.3 \\ 0.6 \\ 0.4 \\ 0.9 \\ 1.0 \end{pmatrix}} = \overline{\begin{pmatrix} -0.1 \\ 0.0 \\ -0.1 \end{pmatrix}} = -1/15 \tag{2.21}$$

with numbers:

$$\begin{pmatrix} 1/4 & 0 & 1/4 \\ 1/4 & 1/4 & 0 \\ -1/4 & 1/2 & 1/4 \\ -1/2 & 1/4 & 1/4 \\ 0 & -1/4 & 1/4 \\ 1/4 & -1/4 & -1/2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0.0 + 1/15 \\ 0.3 + 1/15 \\ 0.6 + 1/15 \\ 0.4 + 1/15 \\ 0.9 + 1/15 \\ 1.0 + 1/15 \end{pmatrix} \approx$$

$$\approx \begin{pmatrix} 0.0167 \\ 0.0083 \\ 0.0333 \\ 0.025 \\ -0.025 \\ -0.0083 \end{pmatrix} \tag{2.22}$$

2.4 Artificial tests of the method

Several data-sets based on real values for the single resistor measurements but the higher values artificially constructed by adding and modifying the low values were built to test the algorithm.

2.4.1 Test 1, using perfectly linear data

These values should line up perfectly, $g_{(1,3)} = g_{(1,2)} + g_{(2,3)}$ for example.

$$\begin{pmatrix} 0.002222 \\ 2.015573 \\ 2.210820 \\ 2.232214 \\ 2.325416 \\ 2.324887 \\ 2.017795 \\ 4.226393 \\ 4.443034 \\ 4.557630 \\ 4.650303 \\ 4.228615 \\ 6.458607 \\ 6.768450 \\ 6.882517 \\ 6.460829 \\ 8.784023 \\ 9.093337 \\ 8.786245 \\ 11.108910 \\ 11.111132 \end{pmatrix} \tag{2.23}$$

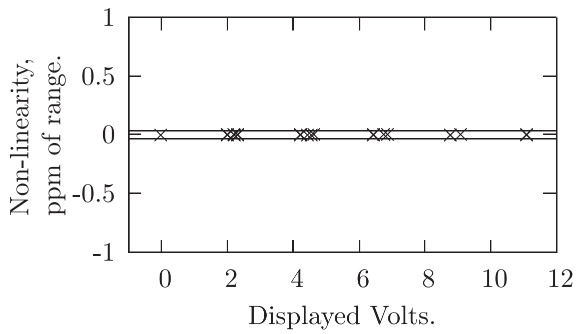

Figure 2.11. Deviations of the fake data, max deviation $6.212 \cdot 10^{-16}$, offset $3.3 \cdot 10^{-17}$

The calculated deviations are illustrated in figure 2.11. The non-linearity of $6.212 \cdot 10^{-16}$ ppm is on level one can expect to get from the limited numerical precision. The offset at $3.3 \cdot 10^{-17}$ V is surprisingly low though.

2.4.2 Test 2, using perfectly linear data with offset

In the second test an offset of 1 V has been added to all measurements. The non-linearity is the same as in test 1 and the offset is accurately calculated to 1 V, see figure 2.12.

$$\begin{pmatrix} 1.002222 \\ 3.015573 \\ 3.210820 \\ 3.232214 \\ 3.325416 \\ 3.324887 \\ 3.017795 \\ 5.226393 \\ 5.443034 \\ 5.557630 \\ 5.650303 \\ 5.228615 \\ 7.458607 \\ 7.768450 \\ 7.882517 \\ 7.460829 \\ 9.784023 \\ 10.093337 \\ 9.786245 \\ 12.108910 \\ 12.111132 \end{pmatrix} \tag{2.24}$$

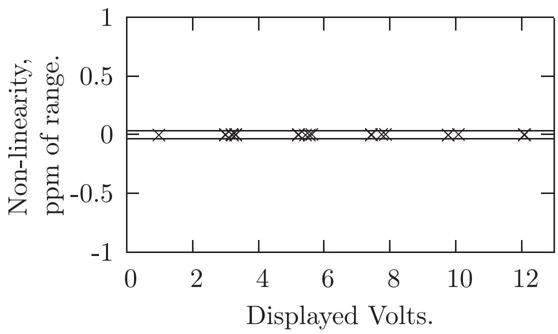

Figure 2.12. Deviations of the fake data and a 1 V offset, max deviation $6.242 \cdot 10^{-16}$, offset 1.0 V

2.4.3 Test 3 using data with a single outlying data-point

In this test the value of $g_{(2,5)}$ has been lowered 100 ppm, 1400μV from 6.458 607 V to 6.457 207 V. The independent non-linearity in this case should be close to 50 ppm. The results given in figure 2.13 show an estimate of 71 ppm.

$$\begin{pmatrix} 0.002222 \\ 2.015573 \\ 2.210820 \\ 2.232214 \\ 2.325416 \\ 2.324887 \\ 2.017795 \\ 4.226393 \\ 4.443034 \\ 4.557630 \\ 4.650303 \\ 4.228615 \\ 6.457207 \\ 6.768450 \\ 6.882517 \\ 6.460829 \\ 8.784023 \\ 9.093337 \\ 8.786245 \\ 11.108910 \\ 11.111132 \end{pmatrix} \tag{2.25}$$

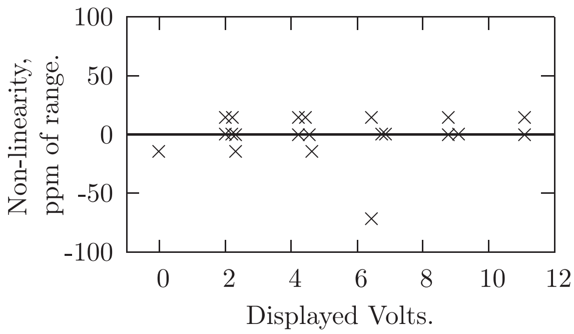

Figure 2.13. Deviations of the fake data and a 100 ppm error, max deviation 71.4 ppm, offset $-59\,\mathrm{aV}$. Independent non-linearity should be 50 ppm.

Chapter 3

Experimental verification

The non-linearity of the DVM-functionality of several multimeters from different manufacturers using varying types of integrating analog-to-digital converters were measured to test the method.

3.1 Equipment

A simple voltage divider using 6 high quality resistors and two rotary switches were manufactured. Only the resistors are of lab-grade, all other components are low cost and/or surplus. None of the components are of low thermoelectric voltage type even though the switches have plastic (low heat conductivity) shafts. All measurement points are accessible either via a direct 4 mm banana plug or via either of two switches. The box is displayed in figures 3.1 and 3.2.

The voltage source used has been either a Fluke 341 or a Fluke 731. The Fluke 731 is a dedicated zener-based voltage reference that outputs a very stable 10 V with a noise level below 1 ppm. The Fluke 341 is a 6 decade voltage calibrator that can output any voltage between 10 µV and 1 kV with noise below 1 mV. All of the measurements performed here using the Fluke 341 show a noise level far below the specification, below 1 ppm.

3.1.1 Limitations

All measurements will be highly affected by noise and interference. **Proper grounding is of highest importance!** Even when using the synchronous Solartron 7075 the reading will differ depending on grounding scheme!

Thermal gradients will affect the measurements, it is not advisable to hold the box while taking measurements and especially not any of the connectors.

The procedure does not depend on the linear/ohmic properties of the resistors. The resistors could be replaced with zener diodes for example. The only requirement is the stability, only very few zener diodes have stability exceeding that of high quality resistors so wire-wound precision resistors seems to be the cheapest option. Neither will thermoelectric voltages within the divider affect the results

Figure 3.1. Picture of the top of the divider/switch-box. The instrument under test is connected to the two sockets closest to the switches. The voltage source is connected over the lower left and right sockets.

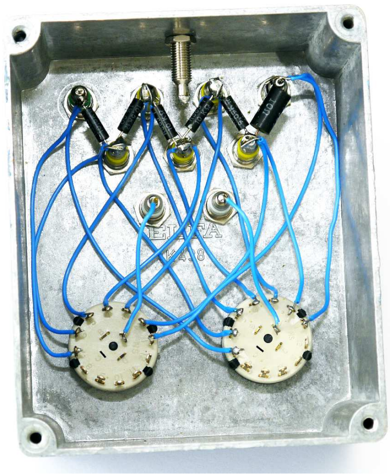

Figure 3.2. View of the inside of the box. All wiring is done as close as possible to the top to minimize thermoelectric effects.

Model	Range	Specified Non-linearity	Measured Non-Linearity
HP 3465A	2 V	n/a	33 ppm
Fluke 8062	2 V	n/a	39 ppm
Keithley 192	20 V	n/a	2.3 ppm
Solartron 7075	14 V	1 ppm	0.20 ppm
Solartron 7065	20 V	n/a	0.55 ppm
Agilent 34461	12 V	n/a	0.67 ppm
Keithley 2010	20 V	n/a	0.39 ppm

Table 3.1. The 8 different multimeters measured.

but differences in the two switches will. The measurements can be properly performed as long as the two switches have identical behavior to each measurement point, only when one switch at a certain position generates a different voltage over the contact compared to the other switch in that same position will an error indistinguishable from a non-linearity be generated.

3.2 Measurements

A number of multimeters has been measured using the method. In practical measurement rounding to the discrete values a DVM can display will limit the linearity to at best 1/2 Least significant digits. For a 6 digit 20 V meter this will be 5 µV or 0.25 ppm.

In all figures the theoretical limit for the measured multimeter is shown with two solid lines.

3.2.1 HP 3465

The HP 3465 is a dual slope 4 digit multimeter with high input impedance on the 2 V and lower ranges. The measurement is close to the theoretical limit as can be seen in figure 3.3. Voltage source used is Fluke 341A set to 1.900 00 V.

3.2.2 Fluke 8062A

The Fluke 8062A is a low cost version of the Fluke 8060 which was one of the first hand-held digital multimeters manufactured. By pressing a special combination of the range switches it is possible to bypass the input attenuator network and it will operate in a high input impedance mode in the 2 V and 200 mV ranges. This multimeter also performs close to the theoretical limit, see figure 3.4. Voltage source used is Fluke 341A set to 1.999 V.

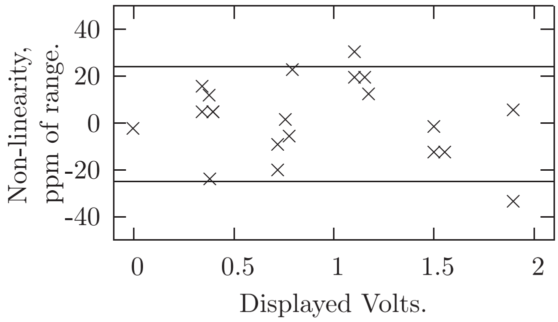

Figure 3.3. Non-linearity for a HP 3465A. The non-linearity is very close to the theoretical limit. Max deviation 33 ppm and offset 26 µV.

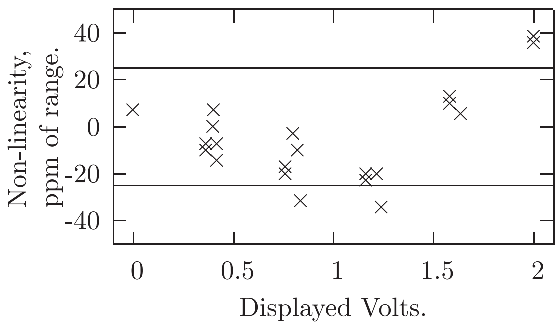

Figure 3.4. Non-linearity for a Fluke 8062A. Max deviation 39 ppm and offset −20 µV.

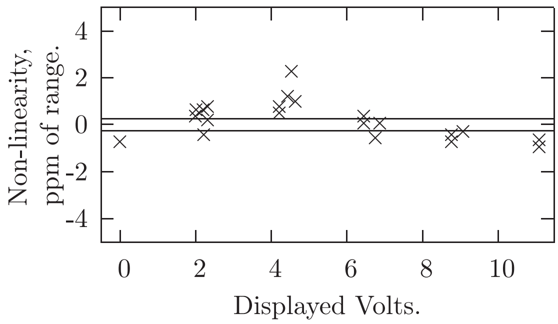

Figure 3.5. Non-linearity for a Keithley 192 in 6.5 digit mode. Max deviation 2.3 ppm and offset 10 µV.

3.2.3 Keithley 192

The Keithley 192 is a system multimeter intended to operate both stand alone and in computerized systems. Being designed in the early eighties it has a very fast conversion rate with 2 updates per second in 6.5 digit mode (10 updates in 5.5 digit mode). To achieve this high rate it uses a two stage integration method where any mismatch between the two stages will introduce some non-linearity. The high-resolution final stage has a resolution 128 times that of the low-resolution stage and has a maximum range about 8 times the quantization-size of the low-resolution stage. This means that a small step in input voltage can shift the conversion to happen more or less in either stage. This is expected to adversely affect the linearity of the conversion, at 2.3 ppm it is almost 10 times the theoretical limit. Moreover, at these low levels any external noise will influence the measurement, especially power-line interference, and the Keithley 192 is not running the conversion phase-locked to the power-line frequency. Measurement data can be seen in figure 3.5. Voltage source used is Fluke 341A set to 11.111 11 V.

3.2.4 Solartron 7075

The Solartron 7075 is a continuously integrating multimeter capable of more than 7 digit resolution at 10 s integration times. It has excellent linearity but lacks automatic detection of zero, it has to be manually zeroed by the operator. The system noise of the 7075 connected to the 731 settles to ±4 µV after several hours. Several measurements using the 7075 have been performed, see figures 3.6 using 341A at 11.111 11 V, 3.7 using 731A, 3.8 using 341A at 11.111 11 V, 3.9 using 341A at 11.111 11 V. The measurements do most likely suffer from noise.

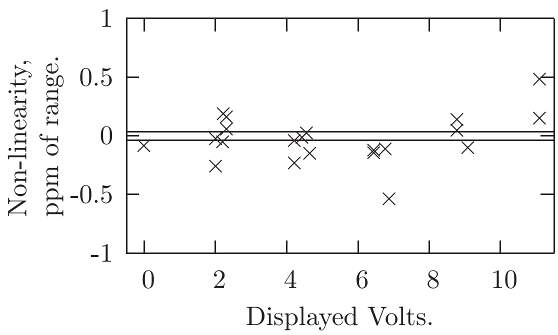

Figure 3.6. Non-linearity for a Solartron 7075 in 10s-mode. Max deviation 0.54 ppm and offset 6.9 µV.

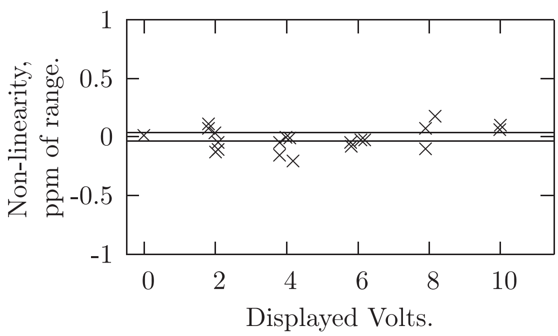

Figure 3.7. Non-linearity for a Solartron 7075 in 10s-mode. This measurements were performed with significantly more care being taken to make sure that each measurement had stabilized before taking a reading compared to figure 3.6. Max deviation 2.0 ppm and offset −3.7 µV.

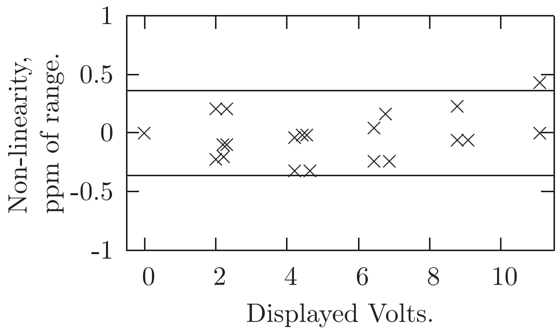

Figure 3.8. Non-linearity for a Solartron 7075 in 1s-mode. Max deviation 0.43 ppm and offset 4.0 µV.

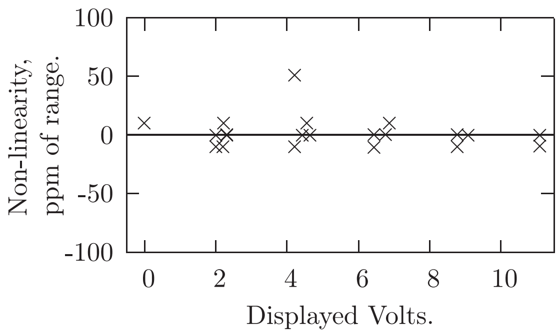

Figure 3.9. Non-linearity for a Solartron 7075 in 1s-mode with a typo of 100 counts, 4.22640 changed to 4.22740. The highest deviation is at the voltage with the typo. Max deviation 51 ppm and offset 4 µV.

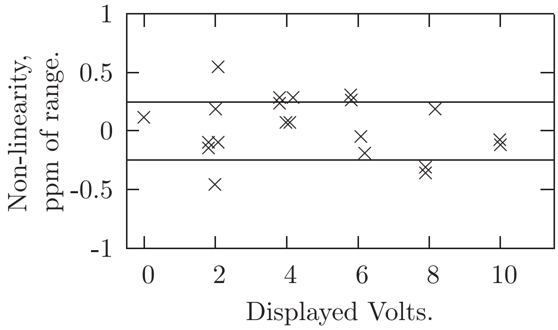

Figure 3.10. Non-linearity for a Solartron 7065 in 1s-mode. Max deviation 0.55 ppm and offset 6.7 µV.

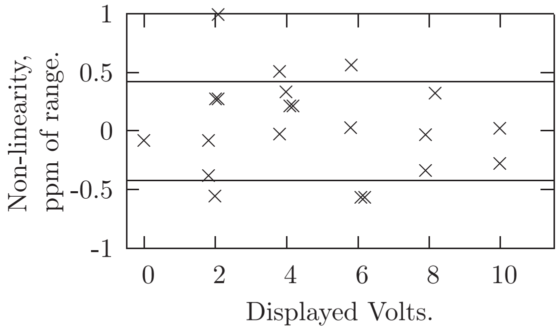

Figure 3.11. Non-linearity for a Agilent 34461 in 10 PLC-mode. Max deviation 0.99 ppm and offset $-2.67\,\mu$V.

3.3 Solartron 7065

The Solartron 7065 is 6.5 digit microprocessor controlled multimeter with a converter very similar to the 7075 with the difference that it will measure zero between each conversion. It is synchronized with power-line frequency. Linearity is close to the theoretical limit as can be seen in figure 3.10

3.3.1 Agilent 34461

The Agilent 34461 is a contemporary multimeter, it is line synchronous. Linearity data in figures 3.11 and 3.12.

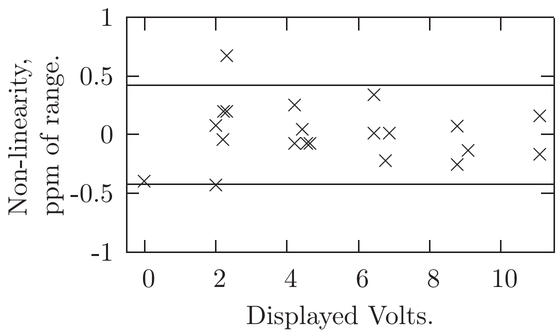

Figure 3.12. Non-linearity for a Agilent 34461 in 100 PLC-mode. Max deviation 0.67 ppm and offset $-1.3\,\mu\mathrm{V}$.

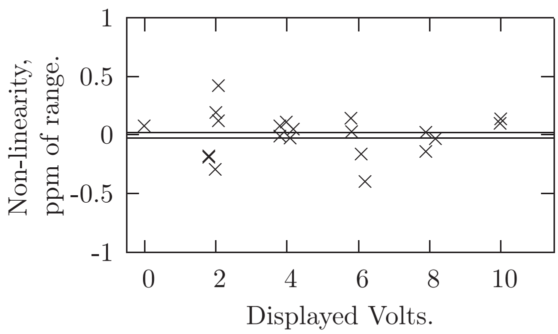

Figure 3.13. Non-linearity for a Keithley 2010. Max deviation 0.42 ppm and offset $0.13\,\mu\mathrm{V}$.

3.3.2 Keithley 2010

The Keithley 2010 is a high precision contemporary multimeter with a full 7.5 digits. The linearity measurements are most likely limited by noise here. The first measurement were performed with a few hours of warmup, figures 3.13, and the second after 24 hours of warmup, 3.14.

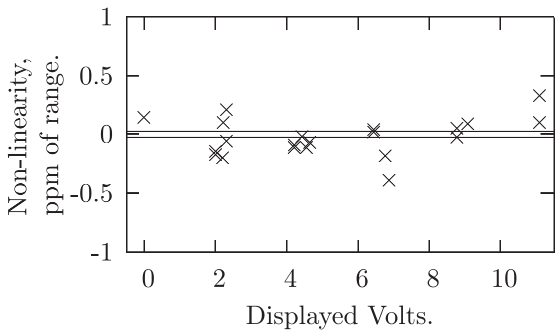

Figure 3.14. Non-linearity for a Keithley 2010. Max deviation 0.39 ppm and offset $-1.4\,\mu V$.

Chapter 4

Discussion

As all tests and measurements performed to reasonable expectations the presented method seems to work. One could argue that the linearity is only tested in a few discrete points but this will be the case for any method, it would take more than 8 years to scan through all possible display values on a Solartron 7075. None of the measurements showed a lower non-linearity than can be expected and those using 7 digit meters will be limited by noise as can be seen in the measurements. Furthermore non-synchronous DVMs usually pick up power-line interference in the 10 µV range when the negative input terminal has a high impedance (more than a few kΩ) to ground. If the presented method is to be used in those cases appropriate buffer amplifiers should be fitted between the divider network and the device under test.

Chapter 5

Summary and Conclusions

A new method for characterizing the linearity of digital voltmeters has been proposed and verified, both theoretically end experimentally. The proposed method is based on measuring voltages from a, compared to current methods, simple voltage divider and solving the under specified equation system generated by applying Kirchhoff's voltage law to the divider. It requires only a stable noise-free voltage source and a handfull of stable noise-free resistors. The components does not need to be of high absolute accuracy, reducing the time for testing by avoiding repeated re-calibration.

The proposed method significantly simplifies the testing procedure and thereby reduces the cost for evaluating linearity compared to the currently used methods. The method does not rely on the tedious calibration procedures required today which saves a lot of time. The only requirement is stability of the voltage divider and the voltage source during the time of the measurement. It does not depend on the linear/ohmic properties of the resistors.

Both the artificial tests and real measurements presented in the report indicates that the proposed method is viable.

Chapter 6

Further work

The relationship between the **e**-vector and actual characteristic of the non-linearity has not been studied in any depth. The equation-system used has been of the minimal form, it is possible to formulate a system with more equations. Even though the extra equations will be linear combinations of those already in the minimal form this might give more and better information about the non-linearity.

Significant simplifications could be made by computer control. The switches should be replaced with low thermoelectric relays and the whole procedure, including averaging of several measurements, relatively easily performed by a program. This will limit the noise and thermal influence introduced by manually rotating the switches and reduce the extreme tediousness of manually reading 8 digits end enter them into calculator many times.

Appendix A

Data formats

A.1 Measurement data-set

$$\mathbf{g} = \begin{pmatrix} g_{(1,2)} \\ \vdots \\ g_{(n-1,n)} \\ g_{(1,3)} \\ \vdots \\ g_{(n-2,n)} \\ \vdots \\ g_{(1,n)} \end{pmatrix} \tag{A.1}$$

Thu number of elements will be $(n-1) + (n-2) + \cdots + 1 = \frac{n(n-1)}{2}$.

A.2 Equation-set

$$\mathbb{F} = \begin{pmatrix} f_{(1,2)} + f_{(2,3)} - f_{(1,3)} \\ \vdots \\ f_{(n-2,n-1)} + f_{(n-1,n)} - f_{(n-2,n)} \\ f_{(1,2)} + f_{(2,4)} - f_{(1,4)} \\ \vdots \\ f_{(n-3,n-2)} + f_{(n-2,n)} - f_{(n-3,n)} \\ f_{(1,2)} + f_{(2,5)} - f_{(1,5)} \\ \vdots f_{(1,2)} + f_{(2,n)} - f_{(1,n)} \end{pmatrix} \tag{A.2}$$

The number of equations will be $(n-2) + (n-3) + \cdots + 1 = \frac{(n-2)(n-1)}{2}$.

A.3 Program code

```
#!/usr/bin/python

import sys
import os.path
from numpy import *
from pylab import *

def build_f(m):
    eqs = m*(m-1)/2
    var = m*(m+1)/2
    f = zeros((eqs, var))
    for n in range(eqs):
        for i in range(m - n - 1):
            f[n*m - n*(n+1)/2 + i, i] = 1
            f[n*m - n*(n+1)/2 + i, i + n*m - ((n-1)*n)/2 + 1] = 1
            f[n*m - n*(n+1)/2 + i, i + (n+1)*m - (n*(n+1))/2] = -1
    return matrix(f)

def calculate_num_res(n):
    return int(round(sqrt(2 * n + .25) - .5))

def calculate_offset(g, f):
    return mean( f * g )

def calculate_e(g, f):
    return linalg.pinv( f ) * f * g

# read input data
fname = sys.argv[1]
lines = [line.strip() for line in open(fname)]
g = matrix(str(lines)).T

# build the matrix
g_len = size(g)
num_res = calculate_num_res(g_len)
f = build_f(num_res)

# calculate the non-linearities
b = calculate_offset(g, f)
e = calculate_e(g - b, f)

# organise output data
a = hstack((g, e))
a= a[argsort(a.A[:,0]),]

# save data to file and print max values
savetxt(str( os.path.splitext(fname)[0] + '.edat'), a)
print "Max deviation: " +  str(max(abs(e))[(0,0)]),
print "Offset: " + str(b),
print fname
```

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