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TECHNOLOGIES

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One Port VNA Calibration, a look under the hood

For a simple one-port calibration, only three error terms are needed, Directivity (D), Source Match (S) and Reflection Tracking (R). These three errors theoretically appear between the VNA and the Device Under Test (DUT). Knowledge of these three error terms allows for correction or calibration of the measurement.

A one-port, 18 GHz Vector Network Analyzer might be used to in an anechoic chamber to measure antenna performance as shown below in Figure 1. Of course proper calibration is required for accurate measurements so determination of the three error terms is essential.

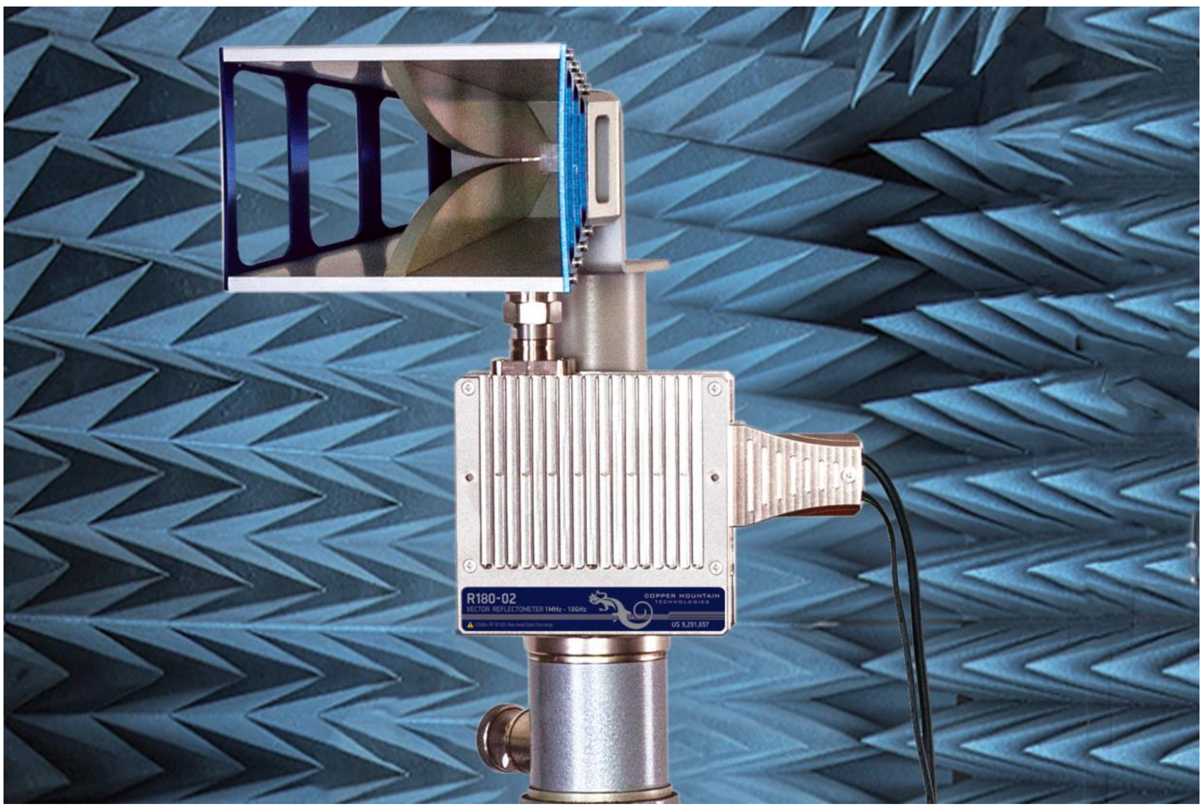

Figure 1 - 18 GHz VNA Horn Measurement

Directivity error is mostly due to leakage in the measurement bridge of the VNA. Some amount of incident signal causes a small erroneous response on the reflection port. A typical raw bridge directivity might be anywhere from 15 dB to 40 dB. The source match represents the error in the source impedance of the VNA. The output source match is never perfect and is made somewhat worse by the cable connector and the test cable. A typical raw source match for a VNA might be 20 dB. The Reflection tracking error is basically the frequency response of the reflected signal from the DUT

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into the VNA and out through the reflection port of the bridge. This includes the loss of the test cable and looks like a small value at low frequencies, increasing to several dB at higher frequencies.

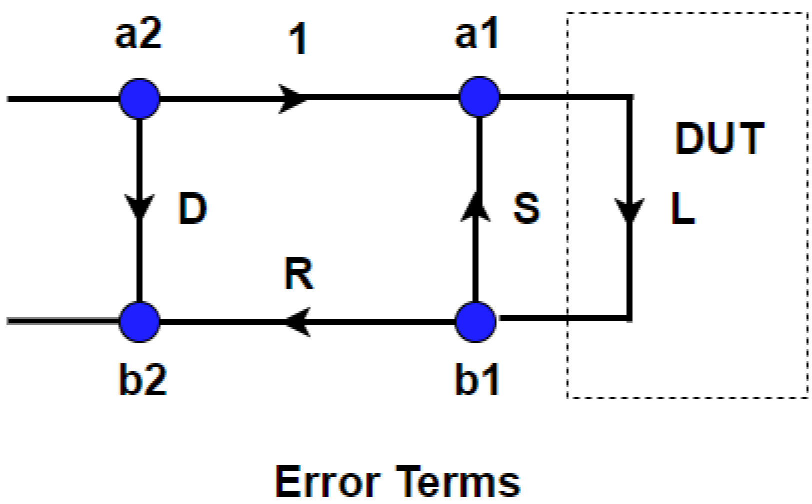

Figure 2- Error Term Flow Diagram

Figure 2 depicts a network signal flow diagram with the three error terms included. A few simple manipulations of this diagram will result in the formula for the measured value as a function of the DUT reflection coefficient and these three error terms.

Note that b2/a2 is our measured reflection coefficient and b1/a1 is the actual reflection coefficient of "L". We want to know b1/a1 while measuring b2/a2. Network flow graph manipulations may be employed to simplify this. For those unfamiliar with the technique, it involves only a few simple rules and an understanding that **the math woks only in the direction of a flow-graph arrow**. For instance, from figure 2, one can say that b2 = R*b1. It is NOT true that b1 = b2/R. This directionality comes about because the flow graph explicitly separates incident and reflected signals. The vectors in Figure 2 all occur on a single coaxial connection but the mathematical relationships treat forward and reverse traveling waves separately.

Network Flow-Graph Rules

The first rule is the "Series Rule".

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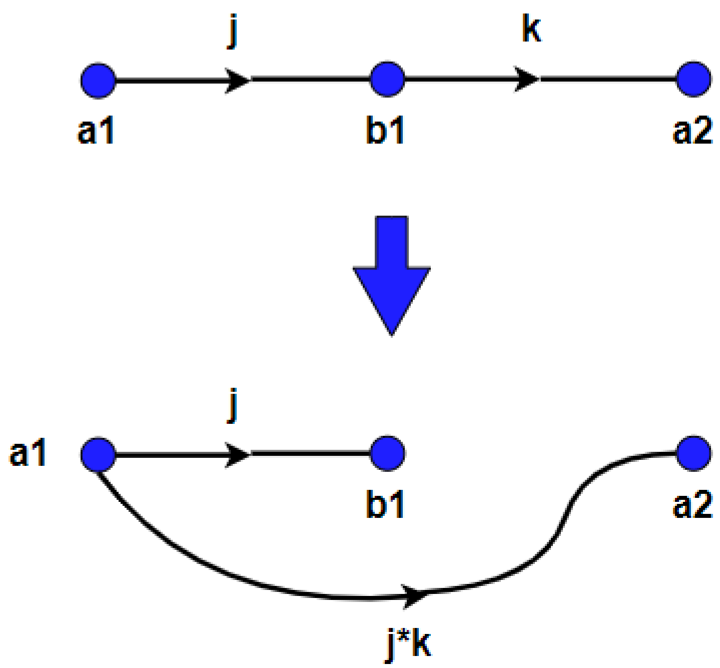

Figure 3 - Series Rule

Because b1 = j * a1 and a2 = b1 * k, then a2 = j * k * a1 and one can break out the connection from a1 to a2. This is useful if a1 is the independent node in the network as it now gives the other two nodes explicitly.

The next rule is the “Parallel Rule”:

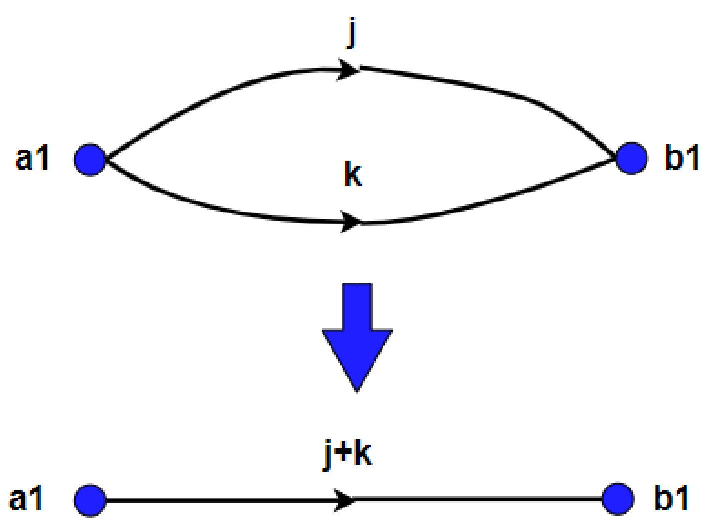

Figure 4 - Parallel Rule

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Because both branches “j” and “k” point in the same direction, their contributions may be combined as shown.

The next rule is the “Self-Loop Rule”:

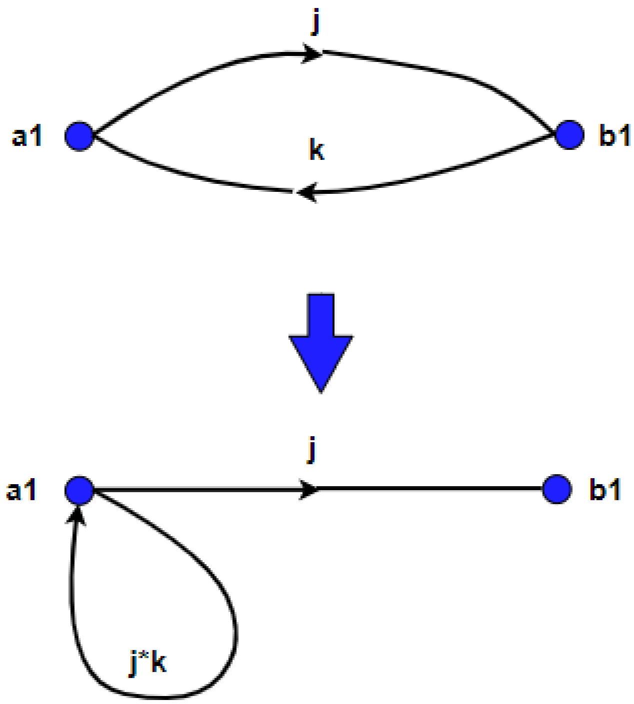

Figure 5 - Self Loop rule

The loop is created by virtue of the Series Rule as the path from a1 to b1 and back may be combined. This may not seem like a useful transform, but one more manipulation is possible. There will always be a network flow that enters the loop in figure 5. The loop may be removed, and its effect applied to that entering path as follows:

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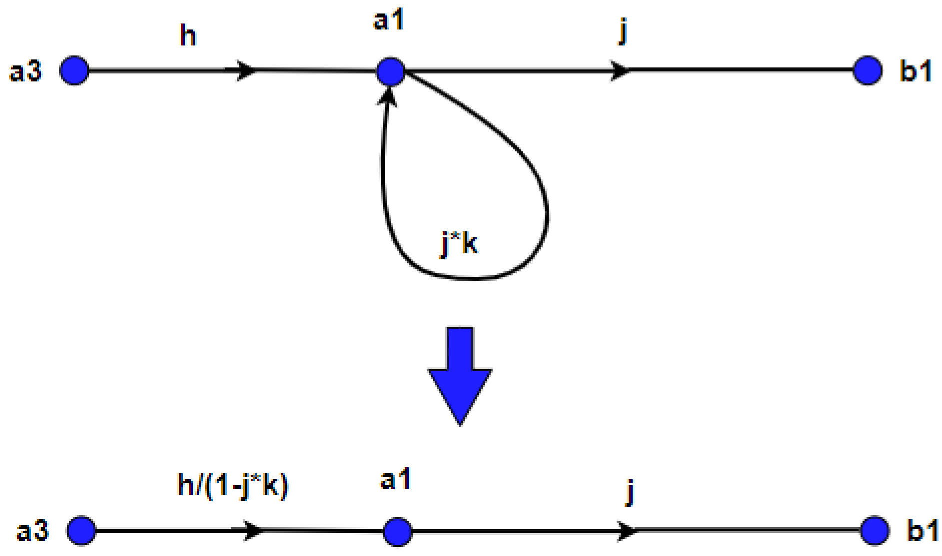

Figure 6 - Loop Elimination

This transform comes from the parallel rule.

$a1 = h * a3 + j * k * a1$ which reduces to $a1 = \frac{h}{1-j*k} * a3$

If the arrow from b1 pointed to the left and also entered the loop, then "j" would have to be replaced by $\frac{j}{1-j*k}$.

These rules may now be employed to simplify figure 2 such that all nodes are explicitly defined by the single independent node "a2". The steps are as follows:

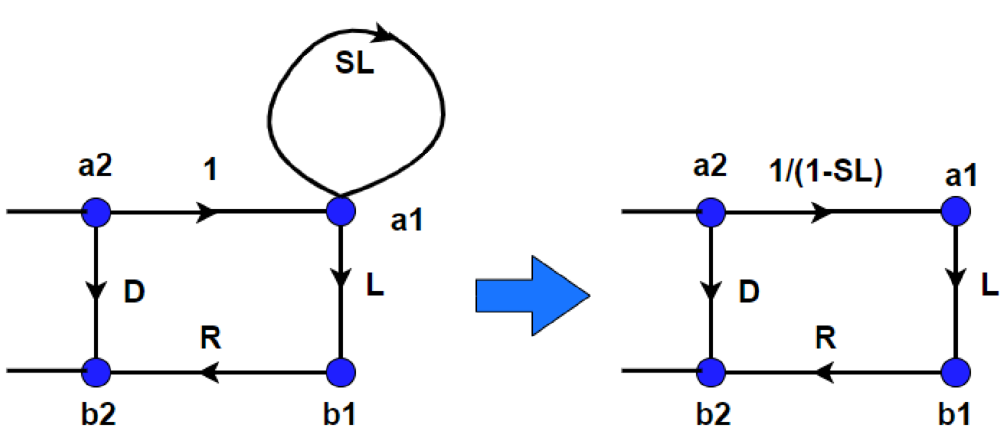

Figure 7 - Loop Rule Eliminates a branch

Followed by the application of series and parallel rules.

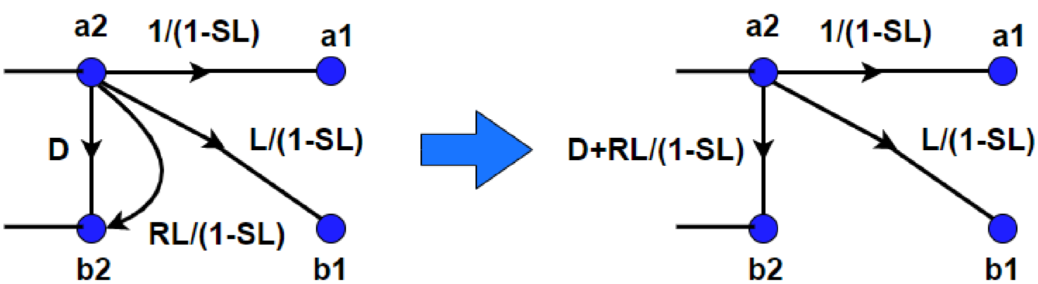

Figure 8 - Series then Parallel Rules applied

Finally, we see that our measured reflection coefficient, $\Gamma_m = \frac{b2}{a2} = D + \frac{RL}{1-SL}$

We can solve this for "L" and obtain: $L = \frac{\Gamma_m - D}{R + S(\Gamma_m - D)}$

This was the result we were looking for.

Now if we know D, R and S we can easily correct our Γ_m to obtain "L" the actual reflection coefficient.

1 Port Calibration

To calibrate, we make three measurements of three known artifacts and solve a system of equations to obtain the D, R and S error terms at each frequency. Optionally, we could make more than three measurements and improve our estimation of the error terms with a "Least Squares" solution. Rather than write out those equations, we'll jump right to a matrix notation which is much cleaner.

Form two matrices "C" and "V".

$$C = \begin{vmatrix} \Gamma_{a1} & 1 & \Gamma_{a1}\Gamma_{m1} \\ \Gamma_{a2} & 1 & \Gamma_{a2}\Gamma_{m2} \\ \Gamma_{a3} & 1 & \Gamma_{a3}\Gamma_{m3} \end{vmatrix} \text{ and } V = \begin{vmatrix} \Gamma_{m1} \\ \Gamma_{m2} \\ \Gamma_{m3} \end{vmatrix} \qquad \text{Eq 1 and 2}$$

Where the Γ_a values are the actuals and the Γ_m values are measured. We must know these actual values a priori. These could be an Open, Short and Load where the actual values are characterized by short delays and parasitic capacitance or inductance as is done in a calibration "kit". The three actual values could also be three shorts with different delays such that the three reflection coefficients are spread around the outside of the Smith Chart over frequency. Any three artifacts with known reflection coefficients may be used as long as the they are sufficiently separated on the Smith Chart at every frequency. If they are not, the matrix calculations will be ill-conditioned and the results unreliable.

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With these two matrices calculate matrix "E"

$$E = (C^H * C)^{-1} * C^H * V \;=\; \begin{vmatrix} E_1 \\ E_2 \\ E_3 \end{vmatrix} \qquad \text{Eq 3}$$

Where "H" is the Hermetian transpose operator, or the matrix transposed with its entries conjugated.

From "E" we can find D, R and S.

$D = E_2$, $S = E_3$, and $R = E_1 + E_2 * E_3$ Eq 4, 5, 6

It so happens that $(C^H * C)^{-1} * C^H$ is a least squares calculation. If our measurements are a little noisy, we can improve our results by making more known measurements. Simply add more rows to C and V! Matrix E will still have three values in the end and the results will be somewhat better in the face of slightly noisy measurements.

Conclusion

The standard one-port three term error model was introduced. Flow-graph manipulation is then used to derive the effects of the three errors on the measured value. Finally, a simple matrix method is shown for easy calculation of 1-Port error correction in order to arrive at calibrated results from the calculated error terms. None of this work is original, but it is educational to pull all the pieces together and demonstrate how they are used.

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