LCR MATH

The complex impedance to be measured is equal to the ratio between the vectorial dimensions $\overline{U_X}$ and $\overline{I_X}$, representing the voltage across the device under test (DUT) and the current flowing through it:

$$\overline{Z_X} = \frac{\overline{U_X}}{\overline{I_X}}$$

Each vector can be broken down into *phase* and *quadrature* components with respect to some fixed reference:

$$Z_X = \frac{V_p + jV_q}{I_p + jI_q}$$

Hence again, using the *series* representation of an impedance $Z_x = R_S + j\,X_S$

$$R_S = \frac{V_p\,I_p + V_q\,I_q}{I_{p^2} + I_{q^2}} \qquad X_S = \frac{V_q\,I_p - V_p\,I_q}{I_{p^2} + I_{q^2}}$$

Some LCR meters go down the analogue route (phase detectors) to obtain the phase and quadrature components of the voltage and current to be measured, the final measurement being performed by an ADC, often of the dual-ramp type for good accuracy, as the DC voltages to be measured are in fact 'contaminated' by a not-inconsiderable residual voltage, if one is looking for a fast response time. The "all digital" method does not suffer from this drawback, and the mathematical operation of discrete Fourier transformation (DFT) makes it possible to obtain the *phase* and *quadrature* values for the voltage (U_p U_q) and current (I_p I_q) from N samples d_i of one period of the signal to be measured

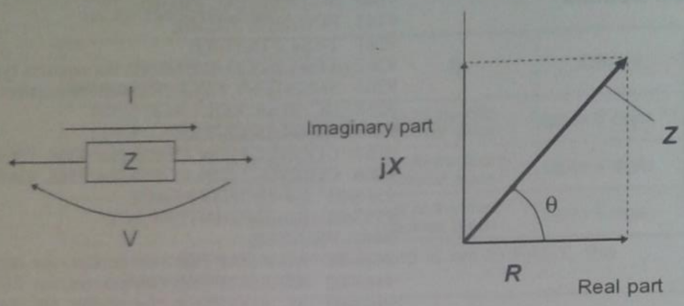
$$U_p = \frac{1}{N}\sum_{i=0}^{N-1} d_i \times \cos\left(\frac{2\pi i}{N}\right) \qquad U_q = \frac{1}{N}\sum_{i=0}^{N-1} d_i \times \sin\left(\frac{2\pi i}{N}\right)$$

This requires just a fast, accurate ADC, and a little bit of calculating power.

Φ – phase angle between voltage and current: $\tan \Phi = |X_s| / R_s$

7.2 Testing Parameters and Calculation Equations

Normal circuit elements etc. are assessed with regard to their characteristics in terms of their impedance Z. The 3522-50 for subjects such circuit components to an alternating current signal at a certain test frequency, measures their voltage and current vectors, and from these values obtains the impedance Z and the phase angle θ. It is then possible to obtain the following quantities from the impedance Z by displaying it upon the complex plane.

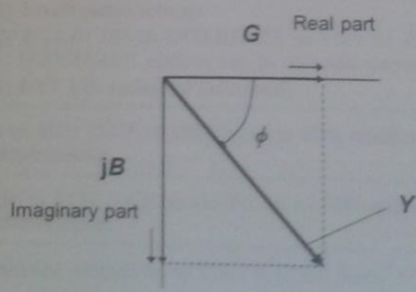


$Z = R + jX$
$\theta = \tan^{-1}\ (X/R)$
$|Z| = \sqrt{R^2 + X^2}$

Z : impedance (Ω)
θ: phase angle (degrees)
R : resistance (Ω)
X : reactance (Ω)
$|Z|$: absolute value of impedance (Ω)

Moreover, it is possible to use the admittance Y, which as a characteristic of a circuit component is the reciprocal of the impedance Z.

By displaying the admittance Y upon the complex plane (just as was done for the impedance Z) the following quantities can be obtained:



$Y = G + jB$
$\phi = \tan^{-1}\ (B/G)$

Y : admittance (S)
G : conductance (S)
B : susceptance (S)
$|Y|$: absolute value of admittance (S)

From the voltage V which is applied between the terminals of the sample under test, the current I which flows through the test sample at this time, the phase angle θ between this voltage V and this current I, and the angular velocity ω which corresponds to the test frequency, the 3522-50 can calculate the following components by using the calculation equations shown:

NOTE

The phase angle θ is shown based on the impedance Z. When measuring based on the admittance, the sign of the phase angle θ must be reversed.

Quantit	Series equivalent circuit mode	Parallel equivalent circuit mode
Z	$\lvert Z\rvert = \frac{V}{I}\left(= \sqrt{R^2 + X^2}\right)$	
Y	$\lvert Y\rvert = \frac{1}{\lvert Z\rvert}\left(= \sqrt{G^2 + B^2}\right)$	
R	$Rs = \mathrm{ESR} = \lvert\lvert Z\rvert \cos\theta\rvert$	$Rp = \left\lvert \frac{1}{\lvert Y\rvert\cos\phi} \right\rvert \left(= \frac{1}{G}\right)^*$
X	$X = \lvert\lvert Z\rvert \sin\theta\rvert$	———
G	———	$G = \lvert\lvert Y\rvert\cos\phi\rvert$ *
B	———	$B = \lvert\lvert Y\rvert\sin\phi\rvert$ *
L	$L_s = \frac{X}{\omega}$	$L_p = \frac{1}{\omega B}$
C	$C_s = \frac{1}{\omega X}$	$C_p = \frac{B}{\omega}$
D	$D = \left\lvert \frac{1}{\tan\theta} \right\rvert$	
Q	$Q = \lvert\tan\theta\rvert \left(= \frac{1}{D}\right)$	

* φ: phase angle of admittance Y ($\phi = -\theta$)

Ls, *R*s, Cs : The measured values of L, C, and *R* in series equivalent circuit mode.

Lp, *R*p, Cp : The measured values of L, C, and *R* in parallel equivalent circuit mode.

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