Analog Active Filters: ”Multiple Feedback” and ”Finite Gain” Topologies - 2^{nd} Order Generic Transfer Functions

The basic transfer functions for second-order multiple-feedback (MFB) and finite gain (aka Sallen-Key) active filter topologies are shown here.

The transfer functions only contain impedances or admittances for the passive components and are not directly suitable for circuit design, but are the basis for deriving the final transfer functions.

The equations presume ideal operational amplifiers (easier today than in the ”741” times). The passive elements can be resistors, capacitors, inductors, or combinations of those to model non-ideal passive components..

All standard second-order filter transfer functions can be derived from the equations. When you insert the Laplace complex component equivalents, they reduce nicely to well-known transfer functions.

This approach gives an intuitive feeling for how elements can be shuffled when designing filters, as opposed to the canned equations generally available.
It also provides an opportunity for using, eg, an LC or an RC lumped combination, or something else as one of the passive elements, possibly creating interesting filter transfer functions.

An example on how to use the equations on a simple filter is shown at the end.

<u>Generally:</u>

W is a symbolic designator (”place holder”) for the passive elements in the schematics and must be substituted with *Y* or *X* depending on your circui

In the equations, *Y* is admittance, *Z* is impedance.

The relationship is:

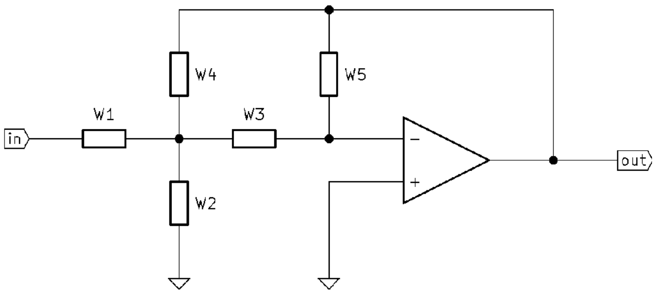
$$Z=\frac{1}{Y} \quad \text{or} \quad Y=\frac{1}{Z}$$

For inserting the basic Laplace notation variables for resistance, capacitance and inductance into the equations, use this table:

Component	**Admittance (*Y*)**	**Impedance (*Z*)**
Resistor	*1/R*	*R*
Capacitor	*sC*	*1/sC*
Inductor	*1/sL*	*sL*

Multiple Feedback Active Filter (MFB)

Schematic:



Transfer function using admittance (Y_n=W_n):

$$\frac{u_{out}}{u_{\text{in}}}=-\frac{Y_1 Y_3}{Y_1 Y_5+Y_2 Y_5+Y_3 Y_5+Y_4 Y_5+Y_3 Y_4}$$

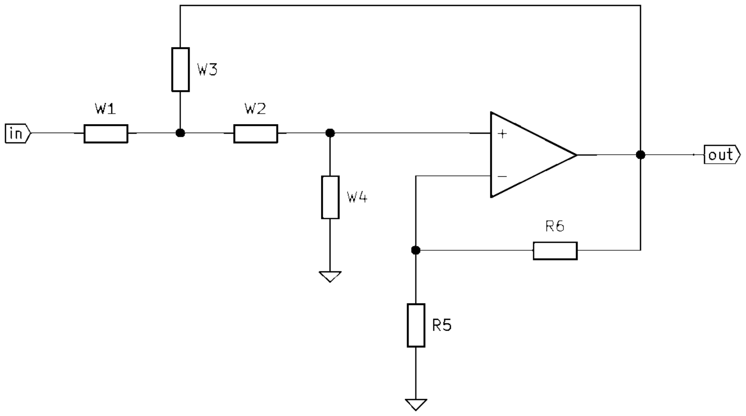
Transfer function using impedance (Z_n=W_n):

$$\frac{u_{out}}{u_{\text{in}}}=-\frac{Z_2 Z_4 Z_5}{Z_1 Z_2 Z_3+Z_1 Z_2 Z_4+Z_1 Z_2 Z_5+Z_1 Z_3 Z_4+Z_2 Z_3 Z_4}$$

Note that this topology inverts the polarity of the input signal.

Finite Gain Active Filter, aka "Sallen-Key"

Schematic:



Note: $K = \frac{R5 + R6}{R5}$ in the equations, where K is the closed-loop gain of the amplifier circuit.

Transfer function using admittance ($Y_n = W_n$):

$$\frac{u_{out}}{u_{\mathrm{in}}} = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_1 Y_4 + Y_2 Y_4 + Y_3 Y_4 + Y_2 Y_3 (1 - K)}$$

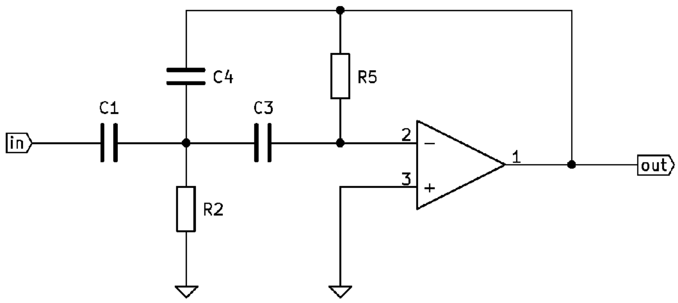
Transfer function using impedance ($Z_n = W_n$):

$$\frac{u_{out}}{u_{\mathrm{in}}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + Z_3 Z_4 + Z_1 Z_4 (1 - K)}$$

Note that amplifier gain (K) is part of the transfer function.

Example:

MFB high pass filter:



Using admittances:

$Y_1 = sC_1$
$Y_2 = 1/R_2$
$Y_3 = sC_3$
$Y_4 = sC_4$
$Y_5 = 1/R_5$

entered directly into the MFB transfer function:

$$\frac{u_{out}}{u_{\text{in}}} = -\frac{Y_1 Y_3}{Y_1 Y_5 + Y_2 Y_5 + Y_3 Y_5 + Y_4 Y_5 + Y_3 Y_4} \quad \text{leads to:} \quad \frac{u_{out}}{u_{\text{in}}} = -\frac{sC_1 sC_3}{\frac{sC_1}{R_5} + \frac{1}{R_2 R_5} + \frac{sC_3}{R_5} + \frac{sC_4}{R_5} + sC_3\, sC_4}$$

Rearranging:

$$\frac{u_{out}}{u_{\text{in}}} = -\frac{s^2 C_1 C_3}{s^2 C_3 C_4 + s\frac{C_1 + C_3 + C_4}{R_5} + \frac{1}{R_2 R_5}} = -\frac{C_1}{C_4} \cdot \frac{s^2}{s^2 + s\frac{C_1 + C_3 + C_4}{C_3 C_4 R_5} + \frac{1}{C_3 C_4 R_2 R_5}}$$

Referencing the normalized high-pass transfer function:

$$F(s) = K \cdot \frac{s^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \quad \text{we see that:}$$

$$K = -\frac{C_1}{C_4} \quad \text{(gain), and} \quad \omega_0^2 = \frac{1}{C_3 C_4 R_2 R_5} \quad \text{and} \quad 2\zeta\omega_0 = \frac{C_1 + C_3 + C_4}{C_3 C_4 R_5}$$

How to proceed from here? Well, you’re the one defining the filter. Gain? your decision. Cutoff frequency? your decision. Damping ratio (ζ) defining filter behaviour? your decision. Calculating component values from the three equations above is easy, as soon as those parameters are fixed.

The admittance/impedance equations are universal.