Analog Active Filters: ”Multiple Feedback” and ”Finite Gain” Topologies - 2nd Order Basic Transfer Functions

This paper lists the basic transfer functions for second-order multiple-feedback (MFB) and finite gain (aka Sallen-Key) active filter topologies.

The transfer functions only contain impedances or admittances for the passive components and are not directly suitable for circuit design, but are the basis for deriving the final transfer functions.

The equations presume ideal operational amplifiers (easier today than in the ”741” times). The passive elements can be resistors, capacitors, inductors, or combinations of those to model non-ideal passive components..

All standard second-order filter transfer functions can be derived from the equations. They seem complex, but when you insert the relevant LaPlace passive element equivalents, they reduce nicely to well-known transfer functions.

This approach gives an intuitive feeling for how elements can be shuffled when designing filters, as opposed to the canned equations generally available.
And it provides an opportunity for using, eg, an LC or an RC combination, or something else as one of the passive elements, possibly creating interesting filter transfer functions.

W is the symbolic designator for the passive elements in the schematics and must be substituted with *Y* or *X*.

In the equations, *Y* is admittance, *Z* is impedance.

The relationship is:

$$Z = \frac{1}{Y} \quad \text{or} \quad Y = \frac{1}{Z}$$

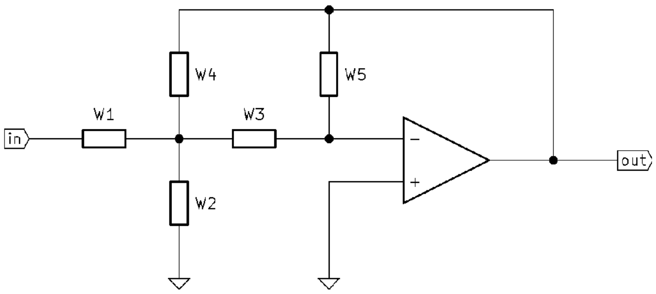
For inserting the LaPlace notation variables into the equations, use this table:

Component	**Admittance (Y)**	**Impedance (Z)**
Resistor	*1/R*	*R*
Capacitor	*sC*	*1/sC*
Inductor	*1/sL*	*sL*

Cheers, Benta.

Multiple Feedback Active Filter (MFB)

Schematic:



Transfer function using admittance (Y_x=$\mathrm{W_x}$):

$$\frac{u_{out}}{u_{\mathrm{in}}}=-\frac{Y_1 Y_3}{Y_1 Y_5+Y_2 Y_5+Y_3 Y_5+Y_4 Y_5+Y_3 Y_4}$$

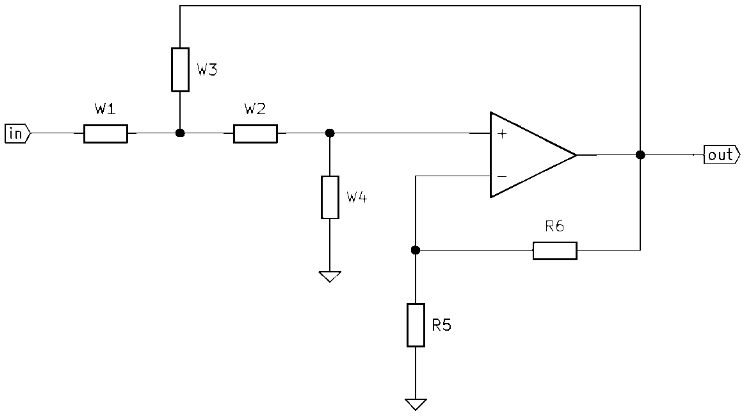
Transfer function using impedance (Z_x=$\mathrm{W_x}$):

$$\frac{u_{out}}{u_{\mathrm{in}}}=-\frac{Z_2 Z_4 Z_5}{Z_1 Z_2 Z_3+Z_1 Z_2 Z_4+Z_1 Z_2 Z_5+Z_1 Z_3 Z_4+Z_2 Z_3 Z_4}$$

Note that this topology inverts the polarity of the input signal.

Finite Gain Active Filter, aka "Sallen-Key"

Schematic:



Note: $K = \frac{R5 + R6}{R5}$ in the equations, where K is the closed-loop gain of the amplifier circuit.

Transfer function using admittance (Y_x=W_x):

$$\frac{u_{out}}{u_{\mathrm{in}}} = \frac{Y_1 Y_2}{Y_1 Y_2 + Y_1 Y_4 + Y_2 Y_4 + Y_3 Y_4 + Y_2 Y_3 (1 - K)}$$

Transfer function using impedance (Z_x=W_x):

$$\frac{u_{out}}{u_{\mathrm{in}}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3 + Z_3 Z_4 + Z_1 Z_4 (1 - K)}$$

Note that gain (K) is part of the transfer function.