

Table I. Capacitance Versus Spacing

Capacitance, Pf	Spacing, Sphere-to-Plane, Cm*
1.....	80
2.....	55
4.....	33
10.....	14

* Sphere-to-cylinder spacing was 56 cm.

Axial Forces and Beam Deflection

The force between the electrodes due to applied voltage could produce a downward deflection of the wood supporting girder and so change the electrode spacing and the capacitance. The amount of this change proved to be significant at the higher voltages. It was determined by making the following measurements:

1. Electrode spacing was changed in several small increments, and the change of capacitance with axial distance, $\Delta C/\Delta X$, was found. The measured capacitance in this case included the capacitance of the sphere to guard as well as the low-voltage plate since both contributed to the force on the girder.

From (1) the force due to a given voltage V was computed from the relation

$$F = \frac{V^2}{2} \cdot \frac{\Delta C}{\Delta X}$$

2. Knowing the forces for the required voltages, small weights in the proper range were added to the normal axial load and the changes produced in the working capacitance were measured; the guard capacitance was excluded from the measurements.

A voltage of about 175 kv gave a correction of 1 ppm; at 350 kv the correction to the 1-pf capacitance was 4 ppm, representing a force equivalent to 190 grams. (See Fig. 6 of reference 1.)

The effect of the axial forces in deflecting the lower working plate was considered inappreciable because of the mass and rigidity of the parts.

Comments

The facilities of the National Bureau of Standards for testing voltage ratio transformers at power frequencies have for many years depended on two things: an oil-cooled, shielded resistor usable to 35 kv and a set of five standard-reference transformers covering voltages to 250 kv. An alternative and independent method of making such determinations, especially above 35 kv, is highly desirable. Experience with the capacitive voltage divider, as herewith reported, shows this

to be a very logical alternative method; it is capable of higher accuracy than the method now used and lends itself readily to higher voltages. That the method is so useful and practical is due largely to development of techniques for the precise comparison of small capacitors.

The capacitor served its purpose very well and afforded useful experience in the use of high-voltage capacitive dividers. Because of its large physical size and its dependence on ambient conditions, a compressed-gas capacitor is being constructed to replace it.

References

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A D-C Wheatstone Bridge for Multi-terohm Measurements with High Accuracy Capability

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Summary: A bridge has been designed that is capable, in principle, of measuring multi-terohm resistance values. For measurements above 10^{10} ohms, the ratio arms of this bridge are formed using a Y circuit. When converted to the equivalent delta, this circuit simulates the large ratio values necessary for measurements above the terohm level. The primary advantage of the circuit is that all resistors within the bridge, none of which is greater than 1 megohm, are wire-wound; this results in inherent accuracy and stability.

RECENT SCIENTIFIC advancements in the development of high-resistance and semiconductor materials have

necessitated the design of instruments capable of measuring the characteristics of these materials. A particularly interesting characteristic is d-c resistance and its reaction to various environments.

The Wheatstone bridge circuit arrangement is probably the most widely used null-type comparison method for the measurement of resistance. Wheatstone bridges, incorporating precision wire-wound resistors, are usually limited to making measurements of less than 10,000 megohms; this limit is imposed by the resistors which form the bridge arms and the insulation which isolates the various

bridge components. Maximum accuracy and stability of the wire-wound standard resistors used in a Wheatstone bridge are obtained when the values of the standards are between 1 and 1,000,000 ohms. Higher valued standards become cumbersome in size and are subject to increasing errors caused by leakage resistance; lower valued standards are subject to errors caused by interconnecting leads and the contacts of plugs or switches associated with the bridge arms.

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The authors wish to express their appreciation to their colleagues at both Bell Telephone Laboratories and the Leeds and Northrup Company; in particular, they wish to thank D. W. Graham of Bell Telephone Laboratories for his help in constructing the prototype described in this paper, Dr. W. G. Amey and F. H. Wyeth of the Leeds and Northrup Company for their critical comments on the manuscript, and H. T. Wilhelm and D. E. Thomas of the Bell Telephone Laboratories for the significant suggestions they contributed.

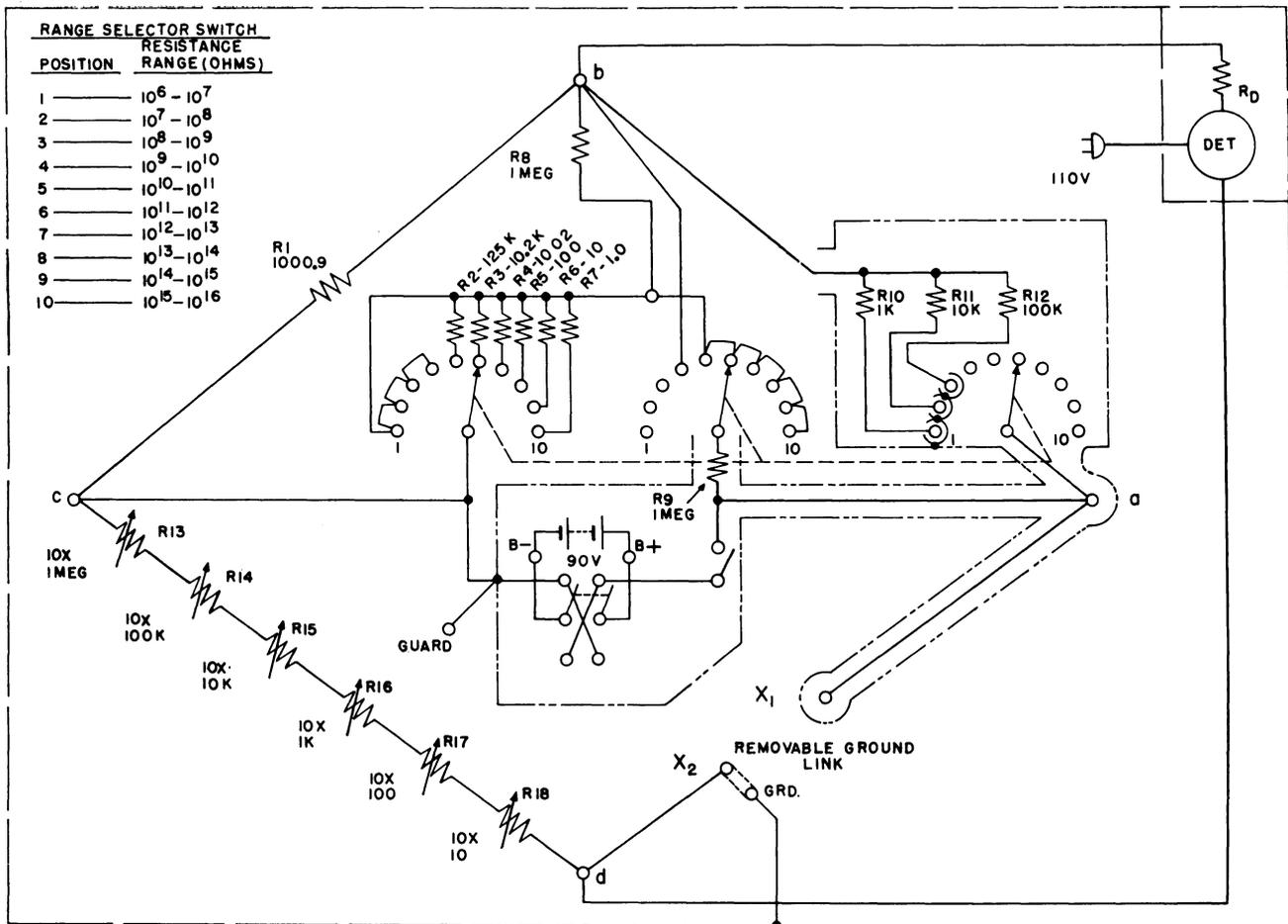


Fig. 2. Circuit diagram of bridge

circuit; equivalent delta values and their effects on the bridge are also given. Fig. 2 shows the complete circuit diagram of the bridge, including the guard circuit.

DESIGN REQUIREMENTS

The multi-terohm bridge described here is designed to measure resistances up to 10^{15} ohms. However, at this level, parallel leakage resistance paths must be considered. Even if the very best insulating materials are used, the magnitude of the insulation resistance could easily approach the magnitude of the measured resistance, particularly under conditions of high humidity. For this reason, it is necessary to provide for guarding throughout the circuit. Although guarding does not eliminate possible leakage paths, it reduces the effect of these leakages on the accuracy of the measurement.

A bridge based on the described principle was built on a prototype basis to solve a specific measurement problem. The bridge, the power supply, and the detector have the following characteristics:

1. *Range:* The bridge consists of a direct reading from 1 megohm to 10^{15} ohms, using

all six decades of the bridge rheostat arm (provision was made to extend the range to 10^{16} ohms, if desired).

2. *Resistor Tolerance:* All resistors have a limit of error within $\pm 0.05\%$ of nominal.

3. *Guarding:* Complete guarding is used to minimize the effect of leakage resistances.

4. *Insulation:* The insulation from circuit to guard has a value of 10,000 megohms, and from guard to ground, 100,000 megohms.

5. *Keys:* A battery reverse key, with a battery on-off key, is used to eliminate unwanted thermal voltages.

6. *Power Supply:* Provision was made for bridge voltages up to approximately 100 volts.

7. *Detector:* The detector has a sensitivity of $\pm 1/2$ microvolt per division, an input impedance of 10 megohms, and a noise level of less than $\pm 1/2$ microvolt.

POWER SUPPLY

In the multi-terohm bridge, a battery is used as the simplest form of power supply because of the ease with which it can be guarded and shielded. Another advantage in using the battery is its compatibility with electronic detectors, which is particularly important since many electronic detectors are adversely affected

by common mode and transverse a-c signals. The particular battery chosen is a Bell System (90-volt) dry battery; it is mounted inside the bridge on a guard plate, which is insulated from the chassis by a sheet of styrene. Dry cells of lower voltage can be substituted for the 90-volt battery when low-valued resistors are being measured.

D-C NULL DETECTOR

The null detector which met the basic requirements for the multi-terohm bridge, is the Boonton Electronics Corporation Model 56-A d-c null detector. This detector combines a 10-megohm input impedance with a $1/2$ microvolt-per-division sensitivity. The 10-megohm input impedance permits the detector to be oriented across the *b-d* bridge corners (Fig. 1) with only a 50% loss in detector sensitivity when the bridge rheostat arm is at the maximum setting of 10 megohms. The manufacturer of this detector has recently offered an alternate design with a 40-megohm input impedance; if all other detector characteristics remain the same, this new design should reduce the loss in detector sensitivity to 20%.

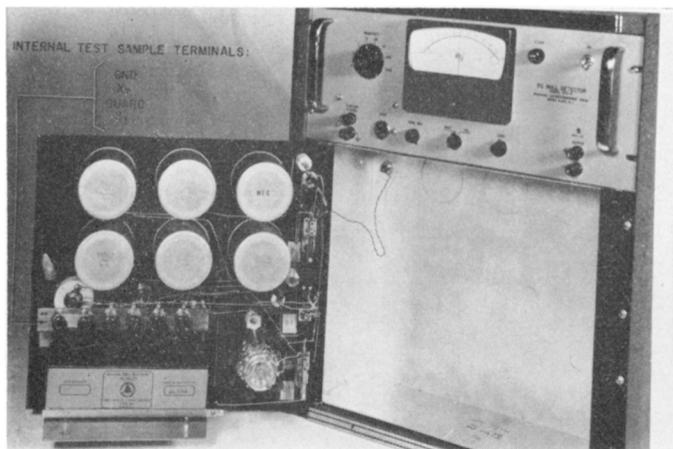


Fig. 3. Internal view of bridge

BRIDGE COMPONENTS

An internal view of the bridge is shown in Fig. 3. The rheostat arm is composed of six resistance decades; the highest decade contains ten steps of 1 megohm each, and the lowest, ten steps of 10 ohms each. All resistors are purchased to a limit of error of $\pm 0.05\%$. With sufficient detector sensitivity, a short-term precision of 10 ppm (parts per million) is theoretically possible over the range of 10^6 to 10^{15} ohms.

Besides the advantage of high accuracy and stability, the relatively low-valued rheostat arm also provides a lower time-constant bridge circuit. The time constant depends on the capacitance associated with the unknown resistor being measured and the resistance of the unknown or rheostat arm, whichever is smaller. In the design described in this paper, the rheostat arm is generally made smaller than the unknown and, consequently, becomes the controlling factor. As explained previously, bridges using high-valued rheostat arms (on the order of 10^{10} ohms or higher) are often adversely affected by, among other things, a long time constant; this can result in considerable annoyance when balancing the bridge.

The ratio arm switch is a special Leeds and Northrup Type 31 silver switch. The switch is constructed with acrylonitrile styrene copolymer as the insulating material, and is mounted on a guard plate to reduce the effect of leakage on the circuit. Each switch contact to which the resistors R_{10} , R_{11} , and R_{12} (Fig. 2) are attached requires complete guarding from the other contacts on that particular switch wafer. This is necessary so that the guard circuit can intercept leakage paths between bridge points a and b (across the switch contact associated with the a point and the contacts with attached resistors R_{10} , R_{11} , and R_{12}). With-

out guarding such leakage would constitute a shunt across the simulated high-valued resistance in the A arm. Guarding is accomplished by painting conductive rings around each contact and making an electrical connection to these rings by placing copper foil between the switch wafers. This method of guarding provides protection against surface leakage only; however, the acrylonitrile styrene copolymer has a sufficiently high volume resistivity so that errors caused by volumetric leakage would be insignificant, compared to sensitivity of measurement.

To provide complete guarding, other bridge components are also of special construction. Power supply keys and critical binding posts are guarded and insulated with the acrylonitrile styrene copolymer. The special shielded external sample holder (Fig. 4) and interconnecting leads are also guarded. Leads to the sample holder are of triaxial construction; the first shield is the guard shield and the second, which is electrically isolated from the first, is the ground shield.

The chassis for the multi-terohm bridge is constructed so that all components (including battery, null detector, and binding posts) are enclosed within a ground shield, a feature that is necessary to minimize effects of unwanted pickup.

Performance Characteristics

Tests on the completed prototype of the multi-terohm bridge confirmed the validity of the basic design principle. Resistors up to 10^{14} ohms were successfully measured. The degree of precision was not as great as expected for the higher resistance values because of excessive noise, as indicated by the detector. For this reason, measurements of higher-valued resistors were not possible. Some of the noise was traced to the interconnecting cable and the internal wiring of the bridge. However, since a full investigation of the source of this noise was not undertaken, it should not be implied that the noise originates predominantly within the detector.

Table II lists various characteristics of the bridge. The maximum calculated resolution is based on the formula developed in the Appendix. The minimum detectable signal is assumed to be $1/2$ microvolt, which represents the maximum noise level of the detector as advertised by the manufacturer.

An example of the sensitivity calculation, in this case for a test resistor of 10^{13} ohms, is as follows:

Equation 7 in the Appendix may be rewritten in terms of p ;

$$p = \frac{V_{det} \cdot A \cdot (R_D + R + B_s)}{E \cdot B_s \cdot R_D}$$

where

p = the proportional change in the nominal setting of the rheostat arm R

V_{det} = the minimum detectable signal by the detector

A = the simulated ratio arm

B_s = the fixed ratio arm

R_D = the internal impedance of the detector

E = the applied bridge voltage

Solving for p gives

$$p = \frac{0.5 \cdot 10^{-6} \cdot 10^9 (10^7 + 10^7 + 10^3)}{90 \cdot 10^3 \cdot 10^7}$$

$$p = 0.011 \text{ or } 1.1\%$$

Under these conditions, therefore, a

Fig. 4. External test sample circuit

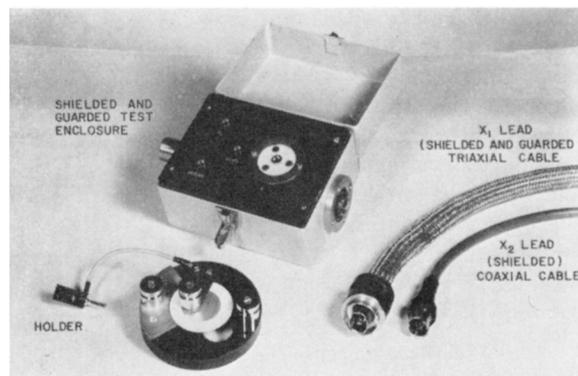


Table II. Various Parameters and Observed Resolution of the Multi-terohm Bridge

Test Resistor (X)	R (Nominal)	A (Nominal)	B _s (Nominal)	Applied Voltage	Maximum Over-all Resolution (Observed)	Maximum Error in Bridge Circuit (Calculated)	Maximum Over-all Resolution (Calculated)
10 ⁶	10 ⁷	10 ²	10 ³	90	± 0.001%	± 0.16%	± 0.001%
10 ⁷		10 ³			± 0.001%	± 0.16%	± 0.001%
10 ¹⁰		10 ⁶			± 0.01%	± 0.26%	± 0.001%
10 ¹¹		10 ⁷			± 0.1%	± 0.26%	± 0.011%
10 ¹²		10 ⁸			± 1%	± 0.26%	± 0.11%
10 ¹³		10 ⁹			± 10%	± 0.26%	± 1.1%
10 ¹⁴	10 ¹⁰	± 100%*	± 0.26%	± 11%			

* Between 1 × 10¹⁴ and 2 × 10¹⁴.

1.1% change from the true balance position of the rheostat arm can be detected.

Calculations for maximum bridge error are based upon the summation of all resistance errors within the bridge, including variations in the B arm resistance resulting from the shunting effects of the wye-delta conversion. Leakage errors are negligible because of the guard circuit and consequently do not affect the accuracy statement.

The maximum bridge error, as recorded in Table II, is determined as follows. The error of the simulated ratio arm resistor A, from equation 1, is

$$A = R_{ao} = R_{ao}(\pm 0.05\%) + R_{bo}(\pm 0.05\%) + \frac{R_{ao}(\pm 0.05\%) \cdot R_{bo}(\pm 0.05\%)}{R_{ao}(\pm 0.05\%)}$$

Since R_{ao} and R_{bo} are each 1 megohm, the first two terms of this equation are insignificant in comparison with the last one. Therefore, the limit of error in A is approximately ±0.15% for values of 10 megohms or more; A is ±0.05% for values less than 10 megohms, because the Y circuit is replaced by a single resistor.

The error in the B_s ratio arm resistance is ±0.06%. This includes ±0.05% for resistor B, plus ±0.01% for the variation in the shunted value caused by R_{bc}.

The possible limit of error of the bridge, contributed by deviations in the resistor elements, is

$$X = R(\pm 0.05\%) \cdot \frac{A(\pm 0.15\%)}{B_s(\pm 0.06\%)}$$

Therefore, neglecting sensitivity, X can be measured to within ±0.26% of the true value.

It should be noted that the limiting factor in the use of the bridge is detector sensitivity. A more sensitive detector would permit more accurate measurement of multi-terohm resistors. If such a detector were available, it might be desirable to perform an over-all bridge check using more stable and noise-free standards than those used in determining

the performance characteristics of the model described in this paper. Such standards could be simulated by a second Y circuit connected to the X terminals and the c corner of the bridge.⁴

Conclusions

The multi-terohm bridge described in this paper and shown in Fig. 5 is a d-c Wheatstone bridge designed to measure resistors from 1 megohm (using all six decades) to above the terohm level. The main advantages of this instrument are as follows:

1. All resistors are wire-wound and none exceeds 1 megohm in value; this eliminates high megohm resistance standards of questionable quality.
2. All bridge resistors can be accurately measured; consequently, their contribution to the performance of the bridge network can be predicted with a high degree of certainty.
3. Measurements are made with optimum speed because of the elimination of long time constants.
4. The bridge is completely shielded and guarded to minimize the effects of stray pickup and leakage resistance.

These features combine to form a Wheatstone bridge inherently capable of measuring multi-terohm resistances with great precision and accuracy and with more stability than is normally found in bridges designed for this purpose.

Appendix. Bridge Sensitivity

This Appendix will show the development of an approximate bridge-sensitivity equation for use with the multi-terohm bridge. It applies to measurements above 10¹⁰ ohms, where high sensitivity becomes important, and is based upon the expression for the departure of rheostat arm R from its balance value in terms of proportional parts.¹ The unbalance voltage across the detector for an unbalance of ΔR in R (see Fig. 1) can be given as follows:

$$V_{det} = E \left[\frac{R + \Delta R}{R + \Delta R + X} - \frac{B_s}{A + B_s} \right] \times \left[\frac{R_D}{R_D + \frac{RX}{R+X} + \frac{AB_s}{A+B_s}} \right] \quad (5)$$

To aid in analysis, equation 5 may be rearranged as follows:

$$V_{det} = E \left[\frac{R \left(1 + \frac{\Delta R}{R} \right)}{X \left(1 + \frac{R}{X} + \frac{\Delta R}{X} \right)} - \frac{B_s}{A \left(1 + \frac{B_s}{A} \right)} \right] \times \left[\frac{R_D}{R_D + \frac{R}{1 + \frac{R}{X}} + \frac{B_s}{1 + \frac{B_s}{A}}} \right] \quad (6)$$

From the bridge-circuit considerations that

$$B_s X = A R$$

$$X \gg R$$

$$A \gg B_s$$

and with

$$\frac{\Delta R}{R} = p$$

equation 6 reduces to

$$V_{det} \cong E \cdot \frac{B_s}{A} \cdot p \cdot \frac{R_D}{R_D + R + B_s} \quad (7)$$

References

1. ELECTRICAL MEASUREMENTS (book), F. K. Harris. John Wiley & Sons, Inc., New York, N. Y., 1959, chapters 7 and 15.

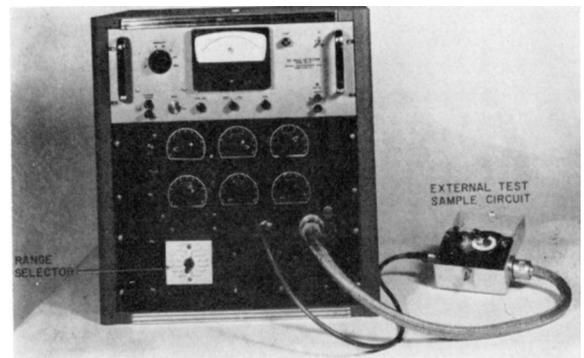


Fig. 5. Wide-range d-c resistance bridge

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4. A PRECISION GUARDED RESISTANCE MEASURING FACILITY, F. H. Wyeth, J. B. Higley, W. H. Shirk, Jr. *AIEE Transactions*, pt. I (*Communication and Electronics*), vol. 77, Sept. 1958, pp. 471-76.

Discussion

H. T. Wilhelm (Bell Telephone Laboratories, Murray Hill, N. J.): This bridge makes ingenious use of a carefully planned Y-delta transformation for extending the precision Wheatstone bridge to higher resistance ranges than previously were possible. The shielded and guarded test enclosure provides the stability which is needed for the precise measurements of compact specimens.

There are, however, some test specimens which will not fit in this test enclosure. For example, cables in a tank are too bulky. Also, the measurement of d-c direct leakage between two terminals in the presence of appreciable ground leakages requires a different technique, even for compact specimens.

Such specimens can be measured on a type of d-c conductance bridge which is like a Wheatstone bridge, but with the standard and one fixed arm interchanged. A particular feature of this bridge is that a zero balance can always be taken to evaluate the residual conductance, if perceptible; a simple subtraction corrects for this residual conductance.

T. E. Wells (National Bureau of Standards, Washington, D. C.): A. H. Scott, in reference 2 of the paper, gives the maximum value of resistance measured as 10^{13} ohms, not 10^{11} ohms as stated in this paper. Since the publication of Scott's paper, this method has been used to measure 10^{14} -ohm resistors with an accuracy of about 0.5%.

Despite certain undesirable features, such as the length of time required to make a measurement of resistance, I have not found any other method of measuring resistors of 10^{12} ohms or higher to be as good.

S. J. Zammataro (Bell Telephone Laboratories, Murray Hill, N. J.): I would like to call attention to the detector sensitivity requirement which is implied in the statement: "With sufficient detector sensitivity, a short-term precision of 10 ppm (parts per million) is theoretically possible over the range of 10^6 to 10^{15} ohms." From the last column in Table II, it is shown that the calculated resolution for measuring 10^{14} ohms is $\pm 11\%$, based on a detector sensitivity of 0.5 microvolt. For this same sensitivity, the calculated resolution for measuring 10^{15} ohms would then be 110%, or approximately 1,000,000 ppm. It follows that, to obtain the stated precision of 10 ppm in measuring 10^{15} ohms, the detector sensitivity would have to be 10 millionths of 0.5 microvolt, i.e., 5 picovolts. One could conclude from this that the authors have advanced the art of the high-resistance bridge to a point which is five orders ahead of the progress in detector sensitivity. However, the referenced statement is better interpreted as another way of saying that the bridge provides a scale which is readable and reliable to 10 ppm from 10^6 to 10^{15} ohms.

H. A. Sauer and W. H. Shirk, Jr.: Mr. Wilhelm refers to the limitations of the

specimen holder described in our paper. This holder was designed for a specific application, but we are confident that a satisfactory sample holder can be designed for practically any application. In connection with cable measurements, Mr. Wilhelm's suggestion of the advantages of the conductance technique is well taken. The method should be most useful for these applications where a measurement in terms of d-c conductance, instead of d-c resistance, is more significant.

With regard to Mr. Wells' comment on the paper by A. H. Scott, we concur that the equipment as originally described by Mr. Scott was capable of measuring resistances up to 10^{13} ohms. However, his paper cites specific tests only up to 10^{11} ohms. At 10^{11} ohms, Mr. Scott quotes an accuracy of measurement of $\pm 0.1\%$ using variable air capacitors calibrated to 0.01% of their maximum range and a timer capable of detecting 0.1 second in the 300 seconds required for a measurement. The accuracy of measurement quoted for a resistor of 10^{13} ohms was 0.2-0.3%, but the paper did not cite the length of time to complete this measurement. Our main concern for the method of measurement proposed by Mr. Scott is the complexity of the techniques which are required in order to obtain accurate results.

Mr. Zammataro's observations concerning bridge sensitivity are correct, and we fully appreciate the present limitations of the bridge described in our paper. We have attempted to separate measurement sensitivity from bridge accuracy, and for this reason, we claim the bridge has a high accuracy capability up to 10^{15} ohms. The use of higher voltage and/or a more sensitive detector would appreciably extend the usefulness of the bridge. The important fact, however, is that the bridge itself is an inherently accurate and stable instrument the upper range is limited only by the lack of a sufficiently sensitive detector.