Wye-Delta Transfer Standards for Calibration of Wide Range dc Resistance and dc Conductance Bridges

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Abstract—Accompanying the improvement of electrical insulating materials, reflected in higher and higher resistivities, is the need for greater confidence in bridge performance with respect to precision and accuracy. To establish such assurance, standards extending to the upper limit of the bridge are required. This paper describes the design and calibration of three-terminal wye (Y) network standards in the teraohm (10^{12} ohms), and multiteraohm (10^{14} ohms) ranges which qualify for this purpose. The Y configuration consists of a geometric arrangement of three resistive components in which one end of each component is connected to a common terminal, while the other ends of the branches constitute the independent terminals of the network. A particular feature is that no solid insulation is used within the assembly to provide mechanical and electrical isolation of the resistance elements. The burden of providing this isolation rests with the insulation of the terminal block of the bridge to which the standard is secured. A further desirable feature is that the resistance elements, none of which exceeds 10^9 ohms, can be calibrated individually without removing, exposing or in any way handling, or disturbing the Y structure. These standards, developed at the Bell Telephone Laboratories and manufactured and calibrated by the Leeds and Northrup Co., have been calibrated also by the National Bureau of Standards. Both organizations report results with a precision of at least one part in one thousand and with an uncertainty not exceeding three parts in one thousand for the 10^{12}-ohm standard. For the 10^{14}-ohm standard the precision on each individual resistor is the same as that for the 10^{12}-ohm standard. However, due to detector sensitivity limitations, the precision of the 10^{14}-ohm Y-network measurement is one part in one hundred with an uncertainty of not more than ten parts in one hundred. These are presently being used to check the performance of wide range precision dc resistance bridges and precision dc conductance bridges; the two types have measurement capabilities to 10^{16} ohms and 10^{-16} mhos, respectively.

BRIDGES of high accuracy and precision, which are capable of measuring resistance well within the multiteraohm range, are required to keep pace with the development of improved dielectric materials. A measure of the stature of such a bridge is the confidence which can be placed in its reliability in the region of very high resistances. To insure high reliability in these resistance measurements it is necessary to establish resistance standards which in themselves enjoy a high degree of confidence. Individual wire-wound resistors greater than 1 MΩ are not convenient because of their inordinately large dimensions. On the other hand, the convenient sized composition resistors above 10^{10} ohms and higher are usually unstable and in the past have been found to be unreliable. Their unreliability, pointed out earlier [1], [2], resides essentially in serious shunting leakage and capacitance effects, hence, long and variable time constants, compounded by high coefficients of resistance with respect to both temperature and voltage, and electrical contact problems within the structure. The Y-network standards, developed at Bell Telephone Laboratories, to be described here require resistances no higher than 10^9 ohms. The low-value resistors are of the metal film deposited type which exhibit high stability and accuracy. The high-value resistors are of the carbon-deposited type and have been specially aged and selected by the manufacturer to insure a high degree of stability. The change in resistance does not exceed 0.05 percent in five years, and they are within 1 percent of their nominal value.

Manuscript received November 8, 1967.
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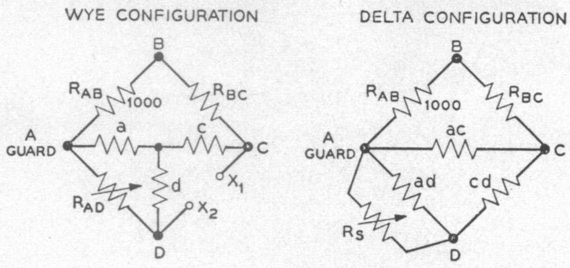
THE *Y* STANDARD

Resistance standards of the Y configuration are made possible because of the electrical equivalence of the wye (Y) and delta (Δ) networks [3]. The virtue of this equivalence resides in the fact that for a properly chosen set of resistors in a Y structure, relatively low-valued resistors connected between two terminals of the Y appear as a simulated resistance orders of magnitude higher across the corresponding terminals of the Δ transformation. This principle has been suggested before [4] to accomplish the same purpose in a bridge whose range did not extend beyond 10^{10} ohms.

The Y network connected to a Wheatstone bridge circuit and the equivalent Δ circuit are shown in Fig. 1 with the Δ-transformation equations. The small individual letters represent the arms of the standard. The small double letters represent the equivalent Δ-circuit components.

The design of a very high resistance standard has for its principal objectives: 1) the mechanical support of the three Y-circuit resistors in such a manner as to eliminate the need for organic insulation between terminals, hence, between components of the Y structure, thus eliminating parallel leakage effects, 2) mechanical support of each resistor component during storage of the standard, and 3) the independent calibration of each component without handling, exposing, or in any way disturbing the network. The latter procedure is a means of detecting a change in the resistance of the standard from its calibration value.



CIRCUIT EQUATIONS

(1) $ad = a + d + \frac{a \cdot d}{c}$ (2) $cd = c + d + \frac{c \cdot d}{a}$

(3) $R_{AD} = \frac{R_{AB}}{R_{BC}} \cdot cd$ (4) $R_S = \frac{ad \times R_{AD}}{ad - R_{AD}}$

Fig. 1. Y-Δ transformation.

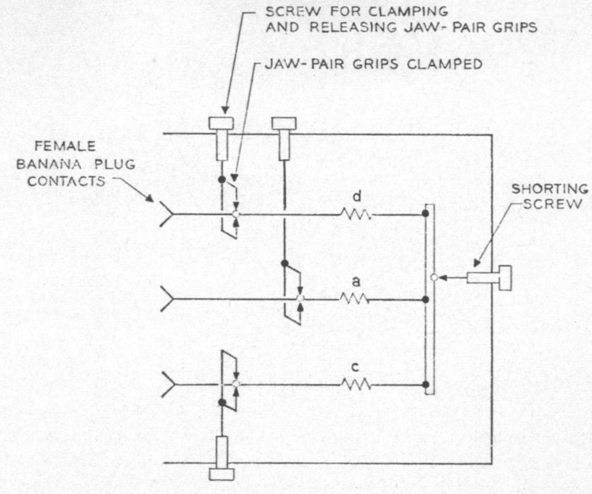
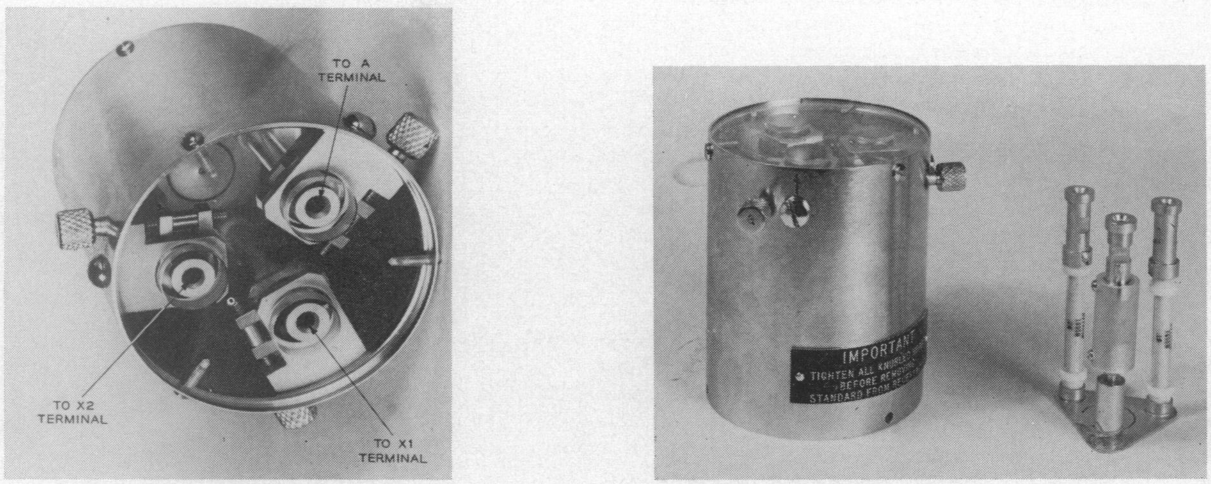


Fig. 2. Schematic of Y standard shown in shorted mode.



(a) (b)

Fig. 3. (a) Y-Δ transfer standard. (b) Y network removed from housing.

A schematic of the Y structure is displayed in Fig. 2. The Y branches, terminated by female banana plugs to provide connection to the a, c, and d terminals of the bridge, are supported by jaw-pair grips. The other end of each resistor is joined to a common bus plate. The grip positions, oriented 120 degrees apart, locate the permanent position of the resistor assembly in its housing. The clamped mode with the shorting screw contacting the bus plate represents the required storage and handling condition of the standard. The complete facility manufactured by the Leeds and Northrup Co., North Wales, Pa., is shown in Fig. 3(a). The Y structure removed from its housing is displayed in Fig. 3(b).

Measurements with Standard

In a typical example, a bridge is employed which is equipped with a triple male plug terminal block, Fig. 4. After the standard is connected to the male banana plug terminals of the bridge illustrated in Fig. 5, the grips and the shorting screw are released. This mechanically and electrically isolates the Y structure from its housing. The network is now supported only by the bridge terminals, illustrated schematically in Fig. 6. Thus, the elements of the structure are air insulated from each other during the bridge calibration measurement. This obviates the necessity of electrical insulation within the standard assembly. After the measurement is performed the network must be gripped before removing the standard from the bridge terminals. At this point, the shorting screw should be adjusted to contact the bus plate.

With an appropriate adapter, and by manipulating the resistor grips and shorting screw in accordance with a prescribed technique, the individual resistors of the standard can be measured. This may be accomplished by employing a calibrated three-terminal wide range bridge of either type described and referred to here, with emphasis on their practical applications, discussed in a more recent paper [5]. Alternatively, a two-terminal resistance measuring facility is also suitable for this purpose. A convenient adapter assembly and carrying case for use with a two-terminal bridge is illustrated in

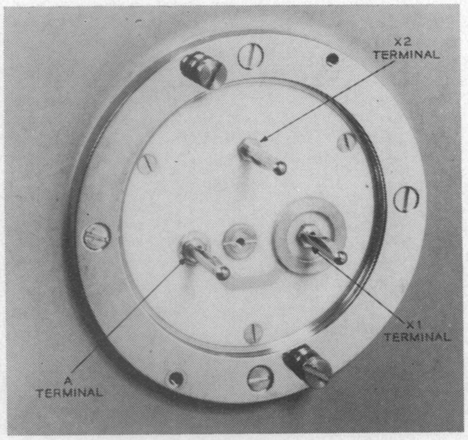


Fig. 4. Terminal block of wide range dc resistance bridge.

Fig. 5. Standard connected to wide range dc resistance bridge.

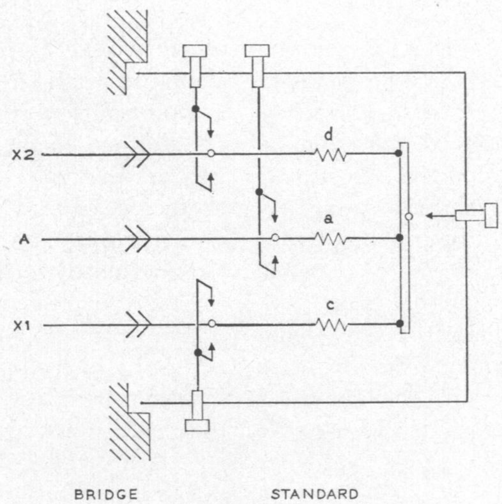
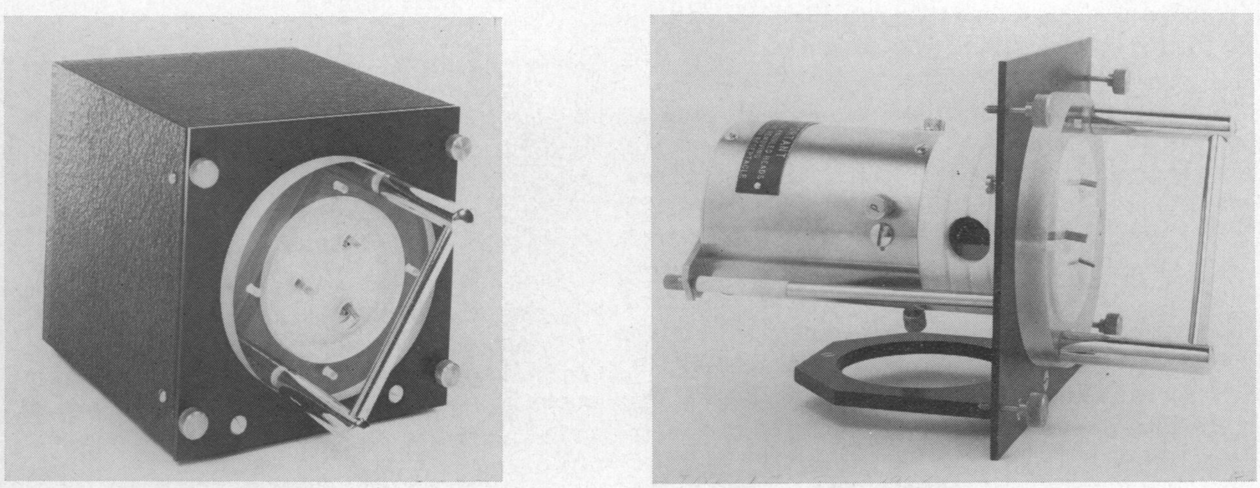


Fig. 6. Schematic of Y standard shown connected to bridge terminals in measurement mode.



(a) (b)

Fig. 7. (a) Carrying case housing standard and accessories for two-terminal calibration. (b) Standard connected to adapter assembly.

TABLE I

LEEDS AND NORTHRUP CO. Y-Δ TRANSFER RESISTANCE STANDARDS

Resistor Designation	Nominal Resistance (Ohms)	NBS Measured Resistance (Ohms)	L&N Measured Resistance† (Ohms)	Measurement‡ Voltage (Volts)	Temperature‖ (°C)	Average Temperature Coefficient of Resistance (Percent per °C)	Uncertainty (Percent)
		Nominal 10^{12} Ohms—Serial No. 1682245					
a component	1×10^6	1.000×10^6	0.999×10^6	0.5	25	−0.002	0.1
c component	1×10^9	0.996×10^9	0.996×10^9	89	25	−0.109	0.2
d component	1×10^9	0.994×10^9	0.994×10^9	1.0	25	−0.124	0.2
cd Y-Δ network*	1×10^{12}	0.998×10^{12}	0.996×10^{12}	90	23.9		0.3
cd calculated	1×10^{12}	0.995×10^{12}	0.996×10^{12}	—	23.9		0.3
		Nominal 10^{14} Ohms–Serial No. 1682244					
a component	1×10^4	0.998×10^4	0.997×10^4	0.5	25	+0.003	0.1
c component	1×10^9	0.998×10^9	0.997×10^9	89	25	−0.127	0.2
d component	1×10^9	1.005×10^9	1.004×10^9	1.0	25	−0.132	0.2
cd Y-Δ network*	1×10^{14}	$0.97\ \times10^{14}$	$1.00\ \times10^{14}$	90	24		10.0
cd calculated	1×10^{14}	$1.00\ \times10^{14}$	$1.01\ \times10^{14}$	—	24		0.3

* Measurements performed employing L&N No. 4233 wide range resistance bridge.

† L&N measurements on a, c, and d components performed employing L&N No. 4232 guarded Wheatstone bridge.

‡ L&N tests indicate voltage coefficient of resistance to be less than 0.0002 percent per volt.

‖ For comparison purposes NBS a, c, and d resistor values have been adjusted to their 25°C equivalents. The cd values have been adjusted to the temperatures at which the NBS cd measurements were made.

Fig. 7(a) and (b). This procedure is desirable for the calibration laboratory possessing only a two-terminal standard bridge. From these measurements the equivalent Δ-resistance values of the standard can then be calculated using the network equations given in Fig. 1. Adapters and specific instructions for performing periodic calibrating measurements may be procured from the manufacturer of the standards.

To illustrate the further flexibility of these Y-Δ standards, they may be used to check the high-value calibration of two terminal wide range bridges. It is merely necessary to connect properly with shielded leads from the adapter terminal block, Fig. 7, to the measurement terminals (X_1 and X_2) and to the A terminal of the bridge.

DISCUSSION OF RESULTS

Table I records the calibration performed by the National Bureau of Standards and the measurements reported by the Leeds and Northrup Co. on the 10^{12}- and 10^{14}-ohm standards. Three-terminal network measurements were made by both the NBS and L&N employing the same facility, namely, the latter's No. 4233 wide range resistance bridge.[1] Two-terminal measurements of the individual resistors were made by L&N employing their No. 4232 resistance measurement facility. The NBS also made two-terminal measurements of the individual resistors. The calculated resistances of the cd arm of the Δ transformation, employing (2) of Fig. 1, are compared in Table I with the measured values. The measurement voltages chosen for the individual components correspond to those which would appear across these components when 90 volts are applied to the bridge circuit. It is significant to point out, however, that the L&N tests indicate that the voltage coefficient of resistance is less than 0.0002 percent per volt. Employing the average temperature coefficient of resistance as indicated in Table I, for comparative purposes NBS a, c, and d resistor values have been adjusted to their 25°C equivalence. For the same reason the measured and calculated cd values of the Δ transformation have been adjusted to the temperatures at which the NBS cd calibrations were performed. The uncertainty estimates, which are exceptionally low, except for the cd measurement on the 10^{14}-ohm standard, take into account systematic and random errors in the calibration procedure. In addition to those inherent in the bridge, and in practicing the technique, these include errors in the measurement of the temperature of the resistor. The large uncertainty (not more than ten parts in one hundred) in the cd measurement on the 10^{14}-ohm standard, with a precision of one part in one hundred, cannot be accounted for on the basis of accumulative errors as determined from the NBS bridge calibration. The possibility of an error of this magnitude, therefore, must be ascribed to the limitation in detector sensitivity, and not to the basic bridge performance. In this connection it is significant to note that the L&N data for the various resistor designations comprise the average of a series of measurements over a period of four months, seven to eleven months prior to the NBS calibration. The individual measurements of this series did not vary from each other nor from NBS measurements by more than 0.1 percent. The unfortunate high uncertainty factor for the 10^{14}-ohm standard points up the necessity for improving detector sensitivity of the facility employed for the direct measurement of these very high-valued Y-Δ resistor networks.

[1] The National Bureau of Standards conducted these measurements only after they calibrated this facility by an independent method.

As pointed out earlier, the voltage coefficient is negligible in affecting the value of the resistor components over the voltage range of the bridge. The temperature coefficient, although rather high for the 10^9-ohm components, need not be taken into account if the bridge check is performed always at the temperature at which the standards have been calibrated.

Finally, while data in the Table I do not provide complete information concerning the long-term aging trend in the networks, tests over an eleven month period indicate good stability. It is, therefore, expected that future periodic recalibrations will establish a high degree of confidence in these standards.

Acknowledgment

The author gratefully acknowledges the careful review given this paper by R. H. Verity and L. C. Fryer of the Leeds and Northrup Co., and by C. Peterson of the National Bureau of Standards. R. H. Verity also aided in the manipulation and tabulation of the data.

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A Systems Analysis of the Cesium Beam Atomic Clock

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***Abstract*—A brief derivation of the transition probability for a two-state atomic system subjected to a phase modulated oscillatory perturbation is presented. This result is used to perform an analysis of a typical cesium beam atomic clock system utilizing a Ramsey (twin cavity) beam tube in order to determine optimum design parameters. It is shown theoretically and experimentally that the optimum modulation frequency for this type of system is higher than has previously been supposed. An additional feedback loop can be incorporated to nearly eliminate cavity phase error.**

I. Introduction

Research in servo-controlled cesium beam atomic frequency standards has been progressing at a rapid rate during the past few years and accuracies of better than 1 part in 10^{11} have been demonstrated by a number of laboratories (a typical system is shown in Fig. 1). Present research is aimed at increasing this figure to 1 part in 10^{13} and better. In view of this objective, a detailed analysis of the cesium beam tube characteristics was carried out so that optimum parameters for the associated electronics system might be chosen. In particular, a comparison between sine-wave phase modulation and square-wave phase modulation of the microwave stimulating signal was made. The optimum modulation frequency for a given beam tube has been determined and appears to be higher than has previously been supposed. In addition, the analysis shows how it is possible to detect and, with the use of a second feedback loop, nearly eliminate the cavity phase difference (which causes a frequency offset) in a Ramsey-type beam tube.

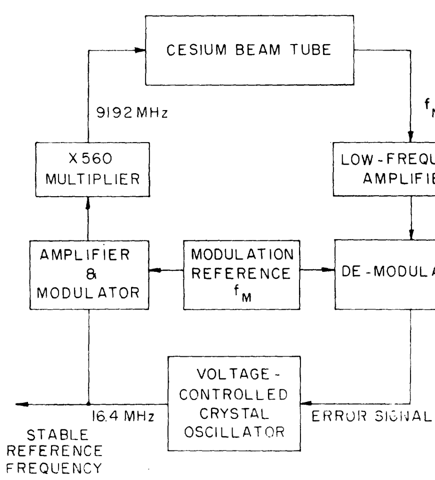

Fig. 1. Typical cesium beam atomic clock system.

Manuscript received February 11, 1968. This work was carried out while the authors were at the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., and was principally supported by the Joint Services Electronics Program under Contract DA28-043-AMC-02536(E).
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