

# A New Measurement Approach for Phase Noise at Close-in Offset Frequencies of Free-Running Oscillators

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**Abstract**—This paper presents a new measurement approach which has been developed as a practical method for quick, accurate and low cost measurements of close-in offset phase noise of microwave free-running oscillators. To overcome the shortcomings of conventional measurement methods, this approach utilizes the injection locking technique to stabilize the free-running oscillator and down-convert the oscillator noise to baseband. Theoretical and experimental studies clearly demonstrate the accuracy, the effectiveness and the flexibility of this measurement technique. The phase noise of voltage controlled oscillators (VCO's) at 2.5 and 9.3 GHz have been measured to verify the new approach.

## I. INTRODUCTION

LOW PHASE NOISE OSCILLATORS are in great demand due to the rapid growth in applications of microwave and RF synthesizers. The oscillator's close-in phase noise is becoming important in improving the overall system performance. In developing high quality, low cost oscillators, one needs to accurately and easily examine the close-in phase noise without the use of expensive equipment. This paper presents a new measurement approach which satisfies this demand.

There are three conventional methods for phase noise measurement: 1) using a spectrum analyzer to measure the power spectrum [1]; 2) using a delay-line or a filter as a frequency discriminator [1]–[3]; and 3) down-converting the phase noise using a clean source (phase detector method) [1], [4], [5]. Each of these measurement methods has its limitation in measuring free-running oscillators due to the high level of long-term frequency instability of the oscillators. A spectrum analyzer can only be used to measure the phase noise at offset frequencies higher than 10 or 100 kHz for most low  $Q$  free-running oscillators. Using a delay-line as a frequency discriminator is a simple and effective approach for observing near carrier phase noise, however, a very long, low loss cable is usually needed to achieve a low system noise floor, which makes it impractical in many applications [1], [4]. In addition, it is difficult to change the length of a delay-line to accommodate different levels of frequency stability of oscillators [4], [6]. The phase detector method is convenient and flexible in terms of system noise floor for phase noise

measurements at different offset-carrier frequencies. However, this approach requires the synchronization of a clean reference source (a synthesizer) to the oscillator using a phase lock loop (PLL), which is difficult to achieve due to the stringent requirement on the loop filter [1]. Furthermore, it may be difficult to lock the oscillation frequency of a low- $Q$  oscillator due to the limited frequency tracking capability of the PLL system [5]. Therefore, a new measurement approach has been developed to overcome the shortcomings of the above conventional methods.

In the new approach a clean reference source is used to injection lock the oscillator under-test to synchronize the oscillation frequency [7]. Due to the inherent frequency discrimination function of the injection locked oscillator, the oscillator's frequency noise is converted into phase noise, which can be detected by a phase detector. Since one can easily apply injection locking to free-running oscillators, and also alter the injection level, this measurement approach actually combines the advantages of the delay-line and phase detector methods. It is simple, flexible, reliable in synchronization, and suitable for near-carrier phase noise measurements. For a measurement system using this approach, in addition to a few microwave components, such as an isolator and a coupler, the only specialized equipment needed is a dynamic signal analyzer.

The measurement setup and procedure using the injection locking technique are introduced in Section II. The theoretical basis of this measurement approach, which is based upon the fundamental theory of injection locking, is described in detail in Section III. The measurement system noise floor and error analysis is also discussed in this section. The experimental verification of the new approach is described in Section IV, which includes: 1) phase noise measurements of a 9.3 GHz VCO compared to the results from an HP phase noise analyzer; 2) phase noise measurement of a 2.4 GHz VCO with a buffer amplifier; and 3) using a dual-oscillator method to avoid the use of a reference synthesizer. Section V discusses the immunity of the new approach to the measurement error due to long-term frequency drift in oscillators.

## II. MEASUREMENT SETUP AND PROCEDURE

The phase noise measurement system using the new approach is shown in the conceptual block diagram in Fig. 1, which is described in detail below.

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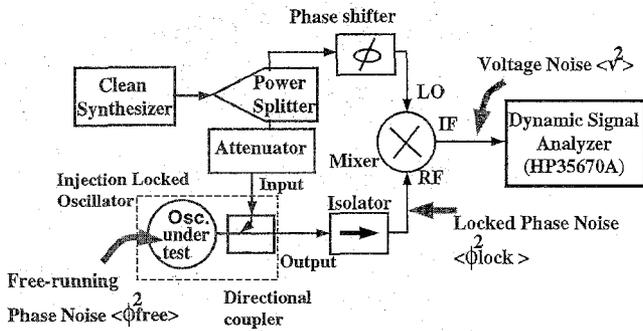


Fig. 1. Phase noise measurement setup using injection locking technique.

- A stable synthesizer is used as a reference source, and its output signal is split into two paths. One signal injection locks the oscillator via a directional coupler; the other goes through a phase shifter to a mixer's LO port to be phase compared to the oscillator signal.
- An isolator prevents any signal due to leakage from the LO to RF ports in the mixer from reaching the oscillator; and the phase shifter (or a line-stretcher) is used to setup a quadrature phase difference between the signals at the LO and RF ports allowing the mixer to work as a phase detector.
- The phase noise difference between the reference signal and the injection locked VCO signal is detected by the mixer at the IF port as a voltage noise,  $\langle v^2 \rangle$ , which is measured by a dynamic signal analyzer.
- The calibration needed to provide the actual phase noise value is a two-step procedure.

Step I) Measure the injection locking range  $\Delta\omega$ , and tune the reference frequency to the center of the locking range (the injection locking range is defined as the total frequency tracking range of the oscillator).

Step II) Measure the phase detector gain,  $G_p$ , by tuning the phase shifter.

- The phase noise of the free-running oscillator  $\langle \delta\phi_{\text{free}}^2 \rangle$  is then retrieved from the measured voltage noise at the dynamic signal analyzer,  $\langle v^2 \rangle$ , by using the following expression for offset carrier frequencies within the injection locking range

$$\langle \delta\phi_{\text{free}}^2(\Omega) \rangle = \frac{1}{G_p^2} \frac{\Delta\omega^2}{4\Omega^2} \langle v^2(\Omega) \rangle. \quad (1)$$

The detailed supporting theory for the measurement is presented in the following text.

### III. THEORY

As it is shown in Fig. 1, the oscillator is injection locked by the reference source to provide the necessary frequency synchronization to allow the phase noise measurement at low offset carrier frequencies. Due to the injection locking from the reference source, for offset carrier frequencies smaller than the locking range, the phase noise detected by the mixer represents the phase noise information of the locked oscillator, but not the free-running one. Only the free-running phase noise at the

frequencies above the locking range can be directly detected by the mixer. If one follows the same approach used the in phase detector method with PLL: using the mixer to directly measure the phase noise outside the injection locking bandwidth, the injection locking range has to be small enough to allow the measurement at close-in offset carrier frequencies. However, such a locking range would be too small to overcome the oscillator's frequency drift and maintain the synchronization for most low- $Q$  oscillators. This is the same dilemma faced by the conventional phase detector method [1], which limits the phase noise measurement for free-running oscillators at close-in carrier offset frequencies. This problem is resolved in the new approach by retrieving the free-running phase noise from the injection locked oscillator noise within the locking bandwidth. To understand the phase noise relationship between the injection locked and the free-running oscillator, the noise analysis of the injection locked oscillator is introduced next.

When an oscillator is injection locked to a reference at its fundamental frequency, the phase of the locked oscillation signal,  $\Phi_{\text{lock}}$  can be expressed as a function of the free-running frequency  $\omega_{\text{free}}$ , and the injection frequency  $\omega_{\text{inj}}$  [7]

$$\frac{d\Phi_{\text{lock}}}{dt} = \omega_{\text{free}} + \omega_{\text{inj}} + \frac{1}{2} \Delta\omega \sin(\Phi_{\text{lock}} - \Phi_{\text{inj}}). \quad (2)$$

In (2),  $\Delta\omega$  is the injection locking range, which is defined as the total frequency range within which the oscillator frequency is the same as the injection frequency.  $\Phi_{\text{inj}}$  is the phase of the injection signal, and  $\Phi_{\text{lock}}$  is a constant when the oscillator is locked. Thus  $d\Phi_{\text{lock}}/dt$  equals zero, and there is static phase shift between  $\Phi_{\text{lock}}$  and  $\Phi_{\text{inj}}$ ,  $\Phi_{\text{detune}} = \Phi_{\text{lock}} - \Phi_{\text{inj}}$ , which can be derived from (2) [7]–[9]

$$\Phi_{\text{detune}} = \arcsin\left(\frac{2\omega_{\text{detune}}}{\Delta\omega}\right) \quad (3)$$

where  $\omega_{\text{detune}} = \omega_{\text{inj}} - \omega_{\text{free}}$ , which is the detuned frequency of the oscillator by the injection signal. Considering the noise as a small perturbation to the steady-state solution, by linearizing (2) one can relate the phase noise of an injection locked oscillator  $\delta\phi_{\text{lock}}$  to the oscillator's free-running frequency noise  $\delta\omega_{\text{free}}$

$$\delta\omega_{\text{free}} - \frac{d\delta\phi_{\text{lock}}(t)}{dt} = \frac{1}{2} \Delta\omega \cos(\Phi_{\text{detune}}) [\delta\phi_{\text{lock}} - \delta\phi_{\text{inj}}] \quad (4)$$

where  $\delta\phi_{\text{inj}}$  denotes the phase noise in the injection signal. By omitting the relatively small contributions of  $d\delta\phi_{\text{lock}}/dt$  and  $\delta\phi_{\text{inj}}$ , (4) can be simplified as

$$\delta\omega_{\text{free}}(t) = \frac{d\delta\phi_{\text{lock}}}{dt} = \frac{1}{2} \Delta\omega \cos(\Phi_{\text{detune}}) \delta\phi_{\text{lock}}. \quad (5)$$

Equation (5) clearly shows that the phase noise of an injection locked oscillator is approximate proportional to its free-running frequency noise by the injection locking range and the static detuning phase shift. Therefore, the injection locked oscillator can be considered as a frequency discriminator which converts its original free-running frequency noise to the phase noise of the locked oscillation signal. This frequency discriminating function can be conceptually explained using Fig. 2, where the transfer function of an injection locked

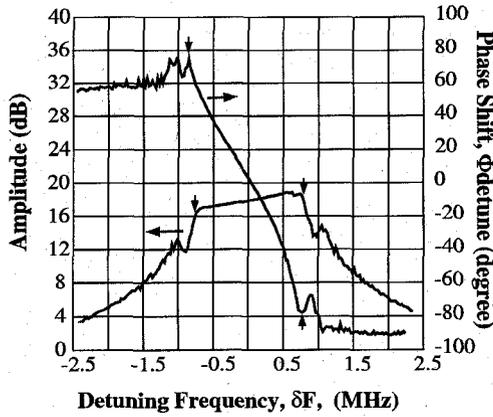


Fig. 2. The transfer function of an injection locked VCO, which is defined as the ratio of the oscillator output to the injection input signal. The detuning frequency is defined as the difference between injection frequency and the original free-running frequency. The free-running frequency of the VCO is 9.3 GHz. The locking bandwidth is about 1.62 MHz, indicated by the two markers.

oscillator is displayed as a function of the detuning frequency. Within the locking bandwidth, the amplitude of the oscillation signal remains flat due to the frequency tracking of the oscillator signal, however, the transfer function phase shift varies approximately from  $+90^\circ$  to  $-90^\circ$  as an arcsine function, as described in (3). The detuned phase shift equals zero when the injection frequency is the same as the oscillator free-running frequency. Clearly, since the phase shift is a strong function of the detuning frequency, any jitter in the detuning frequency of the oscillator will cause jitter in the detuning phase shift.

The power spectrum density of the locked oscillator phase noise can be obtained from (4) assuming that the phase noise of the reference signal is incoherent with the oscillator free-running phase noise [8], [10], [11]

$$\begin{aligned} \langle \delta\phi_{\text{lock}}^2(\Omega) \rangle &= \frac{4\Omega^2 \langle \delta\phi_{\text{free}}^2(\Omega) \rangle + \Delta\omega^2 \cos^2(\Phi_{\text{detune}}) \langle \delta\phi_{\text{inj}}^2(\Omega) \rangle}{4\Omega^2 + \Delta\omega^2 \cos^2(\Phi_{\text{detune}})} \quad (6) \end{aligned}$$

where  $\delta\phi_{\text{free}} = \delta\omega_{\text{free}}/\Omega$ , which is the the free-running phase noise [12].  $\Omega$  is the angular offset frequency for the noise power spectrum. When the locked oscillation signal is phase compared with the reference signal at the mixer, their phase noise difference is detected as voltage noise at the output of the mixer

$$\begin{aligned} \langle v^2(\Omega) \rangle &= G_p \{ \langle (\delta\phi_{\text{lock}} - \delta\phi_{\text{inj}})^2 \rangle \} + \langle \delta v^2 \rangle \\ &= G_p^2 \frac{4\Omega^2 \{ \langle \delta\phi_{\text{free}}^2(\Omega) \rangle + \langle \delta\phi_{\text{inj}}^2(\Omega) \rangle \}}{4\Omega^2 + \Delta\omega^2 \cos^2(\Phi_{\text{detune}})} + \langle \delta v^2 \rangle. \quad (7) \end{aligned}$$

$G_p$  is the phase detection gain of the mixer, and  $\langle \delta v^2 \rangle$  represents the noise floor due to the mixer and the signal analyzer (referred to as the receiver noise). In actual phase noise measurements, the receiver noise floor contribution as well as the phase noise from the reference are negligible compared to the noise of the free-running oscillator. Therefore the oscillator phase noise can be directly retrieved from the

detected voltage noise using the expression

$$\langle \delta\phi_{\text{free}}^2(\Omega) \rangle = \frac{1}{G_p^2} \frac{4\Omega^2 + \Delta\omega^2 \cos^2(\Phi_{\text{detune}})}{4\Omega^2} \langle v^2(\Omega) \rangle. \quad (8)$$

Notice that (1) is a simplified form of (8) for the offset frequencies within the locking range and with a detuning phase shift at zero.

The system noise floor can be obtained from (7) and (8), which is the lowest measurable level of  $\langle \delta\phi_{\text{free}}^2 \rangle$

$$\langle \delta\phi_{\text{floor}}^2(\Omega) \rangle \approx \langle \delta\phi_{\text{inj}}^2(\Omega) \rangle + \frac{1}{G_p^2} \frac{\Delta\omega^2}{4\Omega^2} \langle \delta v^2(\Omega) \rangle. \quad (9)$$

Since the receiver noise is usually white noise, its contribution to the system noise floor is approximately proportional to the inverse of  $\Omega^2$ . If the reference signal is generated by a frequency synthesizer, its phase noise does not increase sharply as a function of offset frequencies within its PLL bandwidth. Therefore, at low offset carrier frequencies, receiver noise contribution dominates the noise floor, whereas, the reference phase noise will become significant at relatively high offset frequencies. Equation (9) clearly shows that the receiver noise contribution can be minimized by reducing the injection locking range, which can be easily controlled by the injection power level. However, there is a lower limit for reducing the injection range, which has to be greater than the frequency drift of the oscillator to maintain frequency synchronization. The following measurement results will demonstrate that this requirement can be easily satisfied for accurate measurements.

For most low- $Q$  oscillators, the phase noise of frequency synthesizers are low enough to provide the necessary noise floor at far-away offset carrier frequencies. Even in the case of measuring an oscillator of which the far-away offset phase noise is lower than that of the available synthesizers, the above injection locking technique can be applied to beat the phase noise of two identical oscillators under-test to retrieve the average phase noise (dual-oscillator method).

In addition to the noise items expressed in (9), the AM/PM noise conversion contributes to noise measurement error when the oscillator's detuned phase shift is far from zero or the phase difference at the RF and LO ports of the mixer deviates from the quadrature position [13]. For most of the phase noise measurements at close-in offset carrier frequencies, such noise contribution is negligible since the AM noise level is much lower than that of the phase noise.

#### IV. EXPERIMENTAL VERIFICATIONS

##### A. Phase Noise Measurement of a 9.3 GHz VCO

A BJT based voltage-controlled oscillator (VCO) at 9.3 GHz was used to validate the measurement approach by comparing the results with that from a HP phase noise analyzer. A M/A-COM BJT with  $0.25 \mu\text{m}$  emitter-width was used as the active device, and the resonator was realized using lumped LC components. The output power of the VCO was 4 dBm, and the injection signal was  $-35$  dBm. An HP8671B frequency synthesizer was used as the reference source, and an HP35670A dynamic signal analyzer was used to detect the

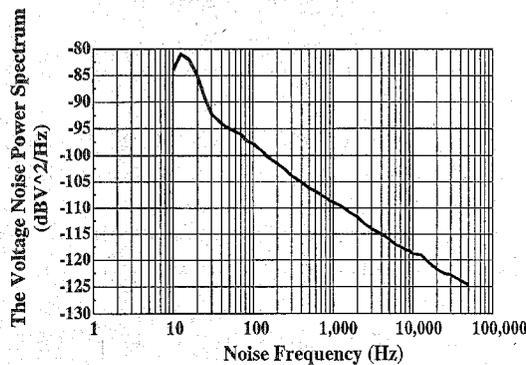


Fig. 3. The power spectrum of the voltage noise detected by the dynamic signal analyzer in measuring the 9.3 GHz VCO. The noise floor of the dynamic signal analyzer,  $\langle v^2 \rangle$  was approximately  $-150 \text{ dBV}^2/\text{Hz}$ .

down-converted noise signal at base-band from 10 Hz to 51 kHz.

Following the measurement procedure described in Section II, the frequency of the reference signal was tuned slowly across the free-running frequency to determine the injection locking bandwidth. The reference frequency was then fixed at the center frequency of the injection locking bandwidth to set  $\Phi_{\text{detune}} = 0$ , and the phase shifter was adjusted to the quadrature phase difference between the LO and RF signals at the mixer. The measured injection locking bandwidth was 1.62 MHz, and the phase detection was 0.102  $\text{v}/\text{radian}$ . The power spectrum of the detected voltage noise for the 9.3 GHz VCO is displayed in Fig. 3. The measured noise power density is approximately a  $1/f$  function of the frequency. This is because the detected noise is proportional to the oscillator frequency noise, which is determined by the  $1/f$  flicker noise of the device at the low frequency [4], [14].

The free-running phase noise of the VCO is then retrieved from the above detected noise using (1), and the results are shown in Fig. 4. The phase noise of the same oscillator measured using an HP phase noise analyzer is also shown in Fig. 4 for comparison. In measuring the VCO's phase noise with the HP phase noise analyzer (HP3047A), both the delay-line and the phase detector methods were attempted. However, only the measurement using a delay-line was carried out successfully due to the difficulty in phase locking the synchronizer to the VCO. The time delay of the delay-line was about 12 ns, and the corresponding system noise floor for this phase noise measurement was approximately  $-75 \text{ dBc}/\text{Hz}$  at 10 kHz.

The comparison clearly shows excellent agreement between the two measurement methods. The slight discrepancies between the two measurement results are mainly due to the different environmental noise sources at the two measurement sites. The system noise contributions in the proposed approach are also displayed in Fig. 4. As described in (9), at close-in offset frequencies ( $< 1 \text{ kHz}$ ), the floor is dominated by the receiver noise of the dynamic signal analyzer, which is 30 to 50 dB lower than the noise of the VCO.

Notice that such low noise floor is a result of the relatively small injection locking range of 1.6 MHz. To achieve the equivalent level of frequency discrimination and noise floor by

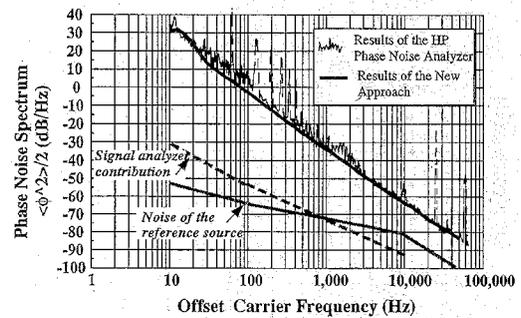


Fig. 4. The measured single-side-band (SSB) phase noise of the 9.3 GHz free-running VCO as a function of offset carrier frequency.

using the conventional delay-line method, one would need a 300 ns time-delay, or approximately a 40 meter low-loss cable at 9.3 GHz, which is relatively impractical. On the other hand, since the long-term frequency drift of this oscillator is below 500 kHz, the injection locking range of 1.6 MHz is more than enough to provide the necessary frequency synchronization during the measurement process. However, using PLL it would be difficult to overcome the 500 kHz frequency drift due to the limitations in the loop bandwidth and the frequency tracking capability of the synthesizer.

#### B. Measurement of Oscillators with Buffer Amplifier

Following the classical theory of injection locking, the strength of the locking force depends on the injection signal level existing in the oscillator's resonator [7], [8]. Since most oscillators have only one output port, the injection signal has to be sent to the oscillator's resonator through the output port. Thus, the injection locking force will be reduced if there is any reverse isolation between the resonator and the output port in the oscillator. In some oscillators, buffer amplifiers are used to prevent load pulling and provide high output power level. Therefore, measuring the phase noise of such oscillators using injection locking may be a concern. However, since this measurement approach only requires a small locking range for frequency synchronization, the leakage of the injection signal from the output to input of the amplifier is usually sufficient to provide the necessary locking range to carry out the measurement without using high power injection. The phase noise of a buffered 2.5 GHz VCO was measured to demonstrate this application.

The oscillator was built using the NEC856 BJT transistor as the active oscillating device and lumped inductor and capacitors on a glass substrate as the resonant elements. The  $Q$  factor of the oscillator is approximately ten. The buffer stage was realized by a 15 dB attenuator followed by a MESFET amplifier to achieve high performance in load pulling. The amplifier provides about 10 dB gain and 8 dBm output power, and the reverse isolation of the amplifier is more than 30 dB.

In measuring the phase noise of this oscillator, the reference signal was injected in to the output port of the amplifier at a power level of  $-4 \text{ dBm}$ . Under such low injection power level, there is no observable change in the oscillator performance in terms of bias current and output power, thus assuring the accuracy and fidelity of the measurement results. The injection

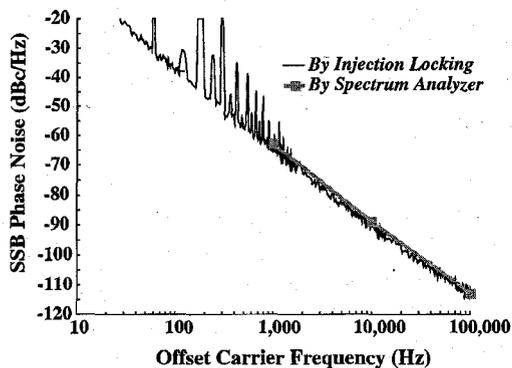


Fig. 5. The SSB phase noise of a 2.5 GHz VCO with one stage MESFET buffer amplifier.

locking range was 161 kHz, which is large enough to cover the oscillator’s frequency drift of 30 kHz at room temperature. The measurement results are depicted as single-side-band phase noise in Fig. 5, and are compared with the measurement results from a spectrum analyzer. Fig. 5 shows a perfect agreement between the two types of measurement results; the maximum discrepancy is less than 1.5 dB within the frequency range from 1 to 100 kHz.

C. Dual-Oscillator Measurements

As it is shown in both above examples, the system noise floor of the new measurement approach at close-in offset carrier frequencies can be adjusted to be low enough for the phase noise measurement due to the flexibility of the injection locking range. However, the phase noise of the reference source dominates the noise floor at offset frequencies far-away from the carrier. Since the commercially available synthesizers do not necessarily have low noise at far-away offsets, one may have difficulty in measuring the low phase noise level of high quality oscillators.

This problem can be overcome by replacing the reference source with another identical oscillator under-test, and retrieving the phase noise by beating the two oscillator signals together. A similar approach has been used in the phase detector method with PLL [15]. It will be shown below that the injection locking approach can also be easily applied to the beating of two identical oscillators for phase noise measurements.

The measurement setup is shown in Fig. 6, which is slightly modified from the system in Fig. 1. Oscillator #1 and oscillator #2 are the two oscillators under-test. Oscillator #2 replaces the synthesized reference source, and there is one isolator placed at its output to prevent any injection force from oscillator #1. Therefore, there is only one-way injection locking from oscillator #2 to oscillator #1. By assuming that the phase noise of the two oscillators are at the same power level and incoherent, one may replace  $\langle \delta\phi_{inj}^2 \rangle$  with  $\langle \delta\phi_{free}^2 \rangle$  in (7) to obtain the oscillator phase noise

$$\langle \delta\phi_{free}^2(\Omega) \rangle = \frac{1}{2} \frac{1}{G_p^2} \frac{4\Omega^2 + \Delta\omega^2 \cos^2(\Phi_{detune})}{4\Omega^2} \langle v^2(\Omega) \rangle. \tag{10}$$

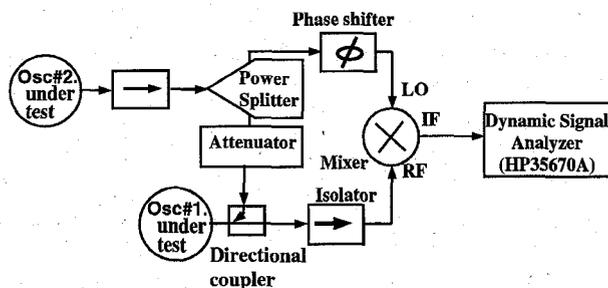


Fig. 6. The measurement setup for phase noise beating of two identical oscillators under-test.

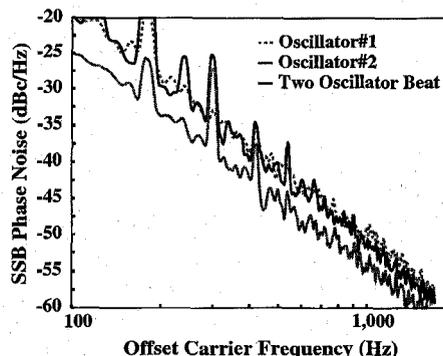


Fig. 7. The measurement results by phase noise beating of the two oscillators under-test.

There is a factor of 1/2 difference between (8) and (10), since the beating noise  $\langle v^2 \rangle$  is the summation of the noise contributions from each oscillator. When there is a slight difference in phase noise between the two oscillators, (10) will result in the average of the phase noise levels of the two oscillators.

The measured phase noise results are shown in Fig. 7 from 100 Hz to 1 kHz, where the phase noise of each single oscillator measured using a synthesizer reference are also displayed for comparison. The phase noise level of the two oscillators are different by 5 dB. Therefore, as expected in (10), the results from the dual-oscillator measurement consistently resemble the average level of the two oscillators, which is about 1 dB lower than oscillator #1 and 4 dB higher than oscillator #2.

V. MEASUREMENT ERROR INDUCED BY FREQUENCY DRIFT

To avoid measurement error, in any noise measurement system, it is important that the system remains unchanged after the calibration in terms of phase detector gain, locking range or time delay and so on. However, during the measurement process, the slow frequency drift of the oscillator will make the system deviate from the calibrated condition, hence causing measurement error. For example, in the delay-line method, the oscillation frequency drift will cause a change in the static phase shift of the delay-line, and such change in phase shift in turn results in a deviation from the quadrature phase relation between the RF and LO ports of the mixer. Clearly any deviation from the calibrated quadrature phase structure in the mixer would reduce its phase detection gain and cause

measurement error [1], [4], [13]. For example, when delay-line method is used for a measurement, if a long-delay-line is used or the measurement process takes a long time, such measurement error will be significant.

Since the injection locked oscillator can be considered a frequency discriminator like a delay-line, one may anticipate similar measurement error in the injection locking approach due to the detuning phase shift caused by the frequency drift. However, by careful inspection of (5), it can be found that the frequency discrimination efficiency of the injection locked oscillator increases when deviates from zero. This increase in the frequency discrimination gain will compensate for the reduction in detection gain in the mixer due to the same nonzero. Such compensation between the oscillator and the mixer phase shift reduces the measurement error due to any oscillator frequency drift, which will be shown in the following analysis.

To retrieve the phase noise using (1), the oscillator detuning phase shift has to be set to zero and the phase difference between the mixer's LO and RF signals is calibrated at  $90^\circ$ . However, during the measurement, there is always a certain amount of frequency drift in the oscillator which induces a nonzero  $\Phi_{\text{detune}}$ . The measurement error can be analyzed by comparing the rigorous expression for measured phase noise under nonzero  $\Phi_{\text{detune}}$  with (1).

After the measurement systems is calibrated, any detuning phase shift in the oscillator will induce the same amount of phase deviation from  $90^\circ$  between the mixer's RF and LO signals. With such deviation, the mixer's response  $v$  to the phase noise  $\delta\phi$  will be [4]

$$v = G_p \cos[\Phi_{\text{detune}}] \delta\phi. \quad (11)$$

Thus, the actual phase detection gain for the phase noise at the mixer is  $G_p \cos[\Phi_{\text{detune}}]$ . Replacing  $G_p$  by  $G_p \cos[\Phi_{\text{detune}}]$  in (8), the rigorous phase noise expression with nonzero  $\Phi_{\text{detune}}$  in the measurement system is

$$\langle \delta\phi_{\text{free}}^2(\Omega) \rangle = \frac{1}{G_p^2} \frac{4\Omega^2 + \Delta\omega^2 \cos^2(\Phi_{\text{detune}})}{4\Omega^2 \cos^2(\Phi_{\text{detune}})} \langle v^2(\Omega) \rangle. \quad (12)$$

When  $\Omega$  is much smaller than  $\Delta\omega \cos[\Phi_{\text{detune}}]$ , the two "cos" functions of  $\Phi_{\text{detune}}$  in (12) cancel each other, resulting in the same expression as (1). This result means that any oscillation frequency drift within the locking range will cause no measurement error, as long as the original phase shift in the oscillator and between the mixer's two ports are set to zero and  $90^\circ$ , respectively.

This advantage of the new measurement approach was demonstrated using a 2.5 GHz VCO. In the measurement, the oscillator's detuning phase shift was set to zero by aligning the reference frequency to the original oscillator free-running frequency. A quadrature phase shift was also set up at the mixer by the phase shifter. Then the reference frequency was intentionally moved away from the free-running frequency to simulate the possible frequency drift and to induce a detuning phase shift. The phase noise was retrieved using (1) at every step of the changing detuning frequency.

The measured phase noise results are shown in Fig. 8 as a function of the frequency drift of the injection locked

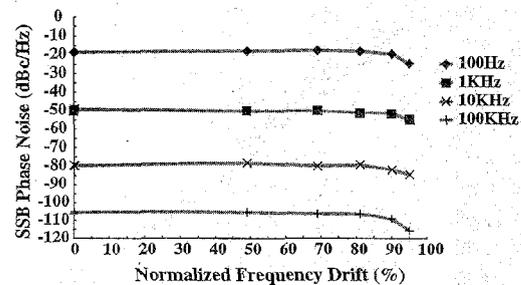


Fig. 8. The phase noise measurement results as a function of the normalized frequency drift of the oscillator. The frequency drift is normalized to the half of the total locking range, which 185 kHz.

oscillator. Clearly for all frequency drifts within 80% of the locking range, the phase noise measurement results at 100 Hz to 100 kHz offsets are very consistent, and the variation in the results are less than 1 dB. This fact shows that the new measurement approach is relatively immune to the measurement error induced by frequency drift of the oscillator. The results in Fig. 8 also demonstrate the good repeatability of the measurement system. The roll-off after 90% drift is mainly due to the quick reduction in the mixer's phase detection gain at large detuning phase shift. The phase noise at 100 kHz offset deviates earlier than other offsets because the injection locking range is only 370 kHz, which is not much larger than the 100 kHz offset frequency. As shown in (12), there is more measurement error at higher offset frequencies.

## VI. CONCLUSION

The above theoretical and experimental results fully demonstrate the advantages of the new phase noise measurement approach using the injection locking technique. Compared to the conventional delay-line and phase detector methods, this approach is simple, accurate, flexible, economical for measurement of close-in offset phase noise of low- $Q$  microwave free-running oscillators.

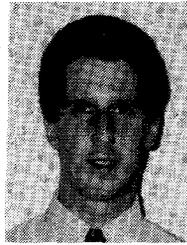
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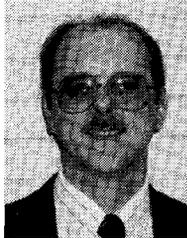
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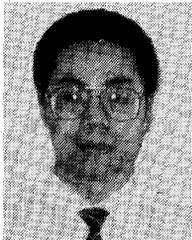
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