Compartmental Analysis of Dielectric Absorption in Capacitors

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ABSTRACT

This paper proposes a method of studying and modeling the dielectric absorption in capacitors. Because of dielectric absorption, the voltage on a charged capacitor partially recovers after momentarily shorting its terminals. The magnitude of this voltage recovery depends mainly on the dielectric material. Dielectric absorption causes errors in some analog applications based on charging and discharging of capacitors, such as sample-and-hold circuits, integrators and active filters. Designing compensation circuits based on models of the dielectric absorption can reduce these errors. This paper presents an analytical method to build a mathematical model of the dielectric absorption, and an equivalent electrical circuit. The method is based on compartmental analysis theory, mostly used in medicine and biology to study the kinetics of substances in biological systems.

1 INTRODUCTION

DIELECTRIC absorption can be observed by momentarily shorting the terminals of a charged capacitor. Starting at 0 V, the voltage on the capacitor rises slowly. The momentary short circuit discharged the conductive plates of the capacitor but some energy still remained stored in the dielectric. This energy recharged the conductive plates, causing the voltage increase. Dielectric absorption causes errors in applications based on charging and discharging of capacitors, such as sample-and-hold circuits, integrators and active filters. Designing compensation circuits based on models of the dielectric absorption can minimize these errors. Dielectric absorption, also called 'soakage' [1], has been known and studied for more than one hundred years [2]. The studies were focused on physical explanation and modeling. The basic model of the dielectric absorption consists of resistor-capacitor time constants connected in parallel with the main capacitor [1], as shown in Figure 1. The number of time constants and the values of the resistors and capacitors are empirically determined by measuring different experimental circuits.

This paper presents an analytic method to build a mathematical model of the dielectric absorption. Based on this model, the paper shows how to determine the number of RC cells and to calculate the values of the resistors and capacitors for a circuit of the type shown in Figure 1. The method is based on the compartmental analysis theory, mostly used in medicine and biology, to study the kinetics of substances in biological systems. The compartmental analysis divides a system in virtual compartments with specific storage capacities and with exponential transfer rate functions. The behavior of the whole system is described by mathematical equations, formed of terms corresponding to each compartment.

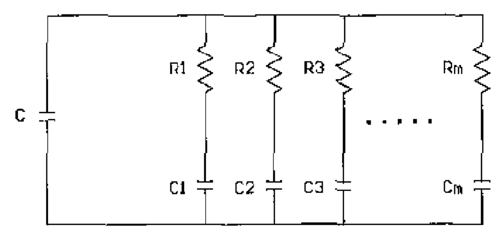

Figure 1. The basic model of the dielectric absorption in capacitors consisting of resistor-capacitor time constants R_1, C_1, R_2, C_2, connected in parallel with the main capacitor C.

2 DESCRIPTION OF THE ANALYSIS METHOD

The proposed analysis method is based on the measurement of the recovery voltage on a fully charged capacitor after momentarily shorting its terminals. A capacitor consists of two conductive plates separated by a dielectric material, as shown in Figure 2(a). When connected to a voltage source, one conductive plate charges positively and the other one negatively. The capacitor remains charged after being disconnected from the voltage source, and can be discharged by shorting its terminals. The voltage on the capacitor is proportional to the amount of charge on the conductive plates. Because of dielectric absorption, the voltage on the capacitor partially recovers after momentarily shorting its terminals. The magnitude of this recovery is lower if the short is maintained for a longer time. Thus, energy still remains in the capacitor after a momentary short. To fully discharge the capacitor, the short needs to be maintained for a long time. This effect of the dielectric absorption can be modeled by adding an energy absorption element in

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parallel with the conductive plates, as shown in Figure 2(b). The electric current charges and discharges the energy absorption element at slow rates. Because of these slow rates, a momentary short discharges only partially the energy absorption element.

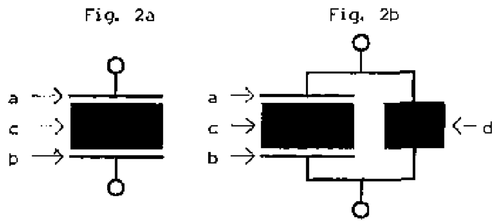

Figure 2. (a) The capacitor is made of two conductive plates, a and b, separated by a dielectric material c. (b) The dielectric can be modeled as an ideal material c in parallel with an energy absorption element d.

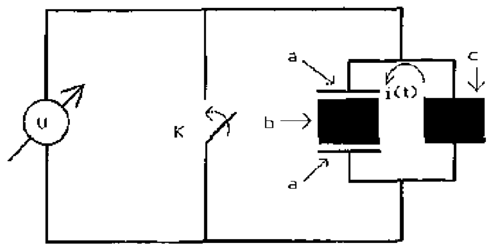

Figure 3. The concept of the recovery voltage measurement. The capacitor is represented by two conductive plates a, separated by an ideal dielectric b and connected in parallel with the energy absorption element c. The switch K can short the capacitor terminals, and the voltmeter V measures the voltage on the capacitor.

The recovery voltage is measured with a voltmeter connected in parallel with the capacitor, as shown in Figure 3. It is assumed that the capacitor is fully charged and the resistance of the discharging path through the switch K is 0 Ω. If the switch K is momentarily turned 'on' for a time ideally equal to zero, the conductive plates will discharge completely while the amount of charge stored in the absorption element will remain unchanged. This charge transfers to the conductive plates at a slow rate, causing the voltage increase. The voltmeter V measures and records this voltage increase as a function $v(t)$. The recording period is assumed to last until the charge transfer ends. Considering an infinite impedance of the voltmeter, no current flows outside the capacitor to the voltmeter. Thus, $v(t)$ represents the voltage variation on the capacitor while being charged with an internal current $i(t)$, as shown in Figure 3. This internal current is assumed to flow from the energy absorption element to the conductive plates, and is calculated using Equation (1)

$$i(t) = C\frac{dv}{dt} \tag{1}$$

where C is the capacitance and v the recovery voltage. The current $i(t)$ is decomposed into a sum of exponential decay terms. Each term represents the current coming from a virtual compartment of the energy absorption element. Thus, the energy absorption element is divided in compartments characterized by energy storage capacitors and exponential transfer functions, similar to adding multiple or distributed relaxation times in dielectric theory. The capacitance and time constant of each circuit are calculated from the $i(t)$ decomposition terms. Based on the calculated values, a mathematical model of the dielectric absorption is built. From the mathematical model it is then shown how to calculate the elements of an electrical circuit model of the type shown in Figure 1.

3 MEASUREMENT SETUP AND DATA RECORDING

The experiment measures and records the recovery voltage on the capacitor after momentarily shorting its terminals. The experiment setup consists of a voltage source U, a voltmeter V, two switches K_1 and K_2, and the capacitor to be modeled C, as shown in Figure 4. The capacitor C is charged at a voltage U with K_1 closed and K_2 open. Then, K_1 opens and K_2 momentarily closes shorting the capacitor terminals. After the momentary short circuit, the voltage rises slowly starting from 0 V, as shown in Figure 5. The recovery voltage is recorded in uniformly timed samples as a series $u(T), u(2T), u(3T), \ldots u(nT)$.

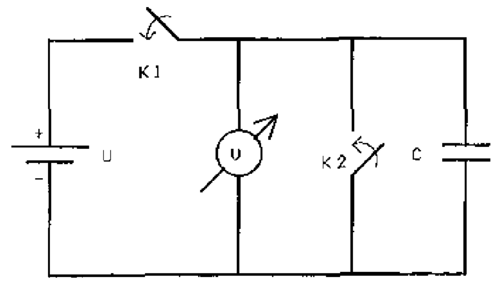

Figure 4. Schematic diagram of the circuit used to measure the recovery voltage on the capacitor.

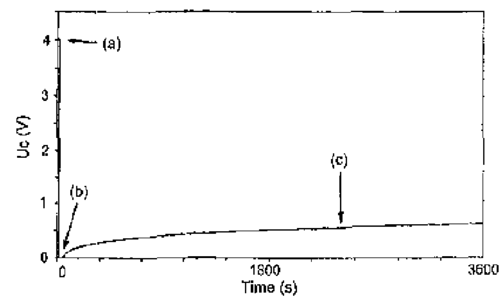

Figure 5. The recovery voltage on the capacitor after momentarily shorting its terminals. The level (a) represents initial charging voltage, (b) represents the momentary short, and (c) represents the voltage variation on the capacitor after the short was removed.

Comparing this setup with the ideal case presented in the previous Section, the impedance of the voltmeter is no longer infinite and the resistance of the shorting path through K_2 is no longer equal to zero. These differences cause errors in the measured values of the recovery voltage. To minimize the errors the following aspects need to be considered.

The input impedance of the voltmeter needs to be very high to minimize the current flowing outside the capacitor. The calculation of the recovery current assumes that current flows only from the energy absorption element to the conductive plates and not outside the capacitor.

Also, the resistance of the shorting path needs to be very low so that the conductive plates will fully discharge during the momentary short.

The capacitor needs to be charged for a long time so that the dielectric, which has a longer time constant, will absorb energy close to its full capacity. Because the time constant of the dielectric is not known at the beginning of the experiment, an arbitrary charging period should be used. To verify that the capacitor was fully charged, the experiment needs to be repeated using a longer charging period. Thus, two models are built for two different charging periods. If the capacitor was fully charged the two models would be identical.

The recovery voltage should be recorded for enough time to cover the longest time constant of the dielectric. Because the longest time constant is not known at the beginning of the experiment, an arbitrary recording period should be used. To verify that the recording period was enough long, the experiment needs to be repeated using a longer recording period. Thus, two models are built for the two recording periods. If the periods are long enough, the two models will be identical.

4 DATA PROCESSING AND MATHEMATICAL MODEL CONSTRUCTION

The recovery voltage $u(t)$ was recorded as a series $u(T)$, $u(2T)$, $u(3T)$, ... $u(nT)$, measured with a sampling period T. The internal current $i(t)$ that recharged the conductive plates is calculated as a series $i(T), i(2T), i(3T), \ldots, i((n-1)T)$, using the discrete form of Equation (1)

$$i(kT) = C\frac{u[(k+1)T] - u(kT)}{T} \qquad k = 1, 2, 3, \ldots (n-1) \tag{2}$$

where C is the capacitance, $u(kT)$ the recovery voltage samples, n the number of samples, and T is the sampling period. The series $i(kT)$ represents the variation of the current $i(t)$ flowing from the energy absorption element to the conductive plates. This current decreases continuously with time and approaches zero corresponding to an equilibrium in the charge transfer. Therefore, $i(t)$ can be represented as a sum of exponential decay currents coming from virtual compartments of the energy absorption element. Each compartment is characterized by storage capacitance and time constant. It is also assumed that the individual currents flow only into the conductive plates and not between compartments. Each individual current is described by an equation of the form

$$i_{j(t)} = I_j \exp\left[-\frac{t}{\tau_j}\right] \tag{3}$$

where I_j is the value of the current coming from the compartment j at time $t = 0$, and τ_j is the respective time constant. The recharging current $i(t)$ is the sum of all individual currents

$$i(t) = \sum_{j=1}^{m} I_j \exp\left[-\frac{t}{\tau_j}\right] \tag{4}$$

where m is the number of compartments. Considering that the capacitances and time constants are different for each compartment, the recharging currents $i_j(t)$ will end successively with time during the recording period. Therefore, it can be assumed that the last section of the $i(kT)$ series represents the current coming from a single compartment. This current decreases exponentially and can be represented on a semi-logarithmic graph paper as a straight line. Thus, representing the $i(kT)$ points on a semi-logarithmic graph paper, there is a set of points at the end of the graph that can be approximated with a straight line. Therefore, the asymptotic tangent to the end of the graph represents the current coming from the last compartment. This current decreases exponentially and is described by Equation (3). The parameters I_j and τ_j of Equation (3) are calculated from the slope and the intercept of the asymptotic tangent.

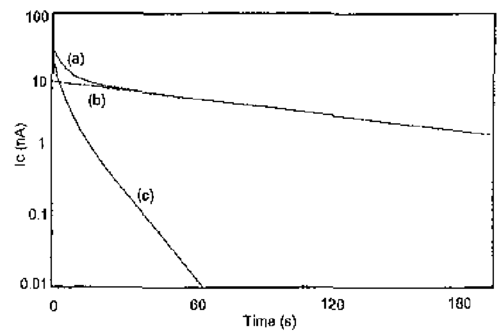

Figure 6. Semi-logarithmic plot showing exponential decay functions as straight lines. The line corresponding to the current coming from a single compartment (b) is subtracted from the curve representing the total current (a). The resulting curve (c) represents the discharging current from the rest of compartments.

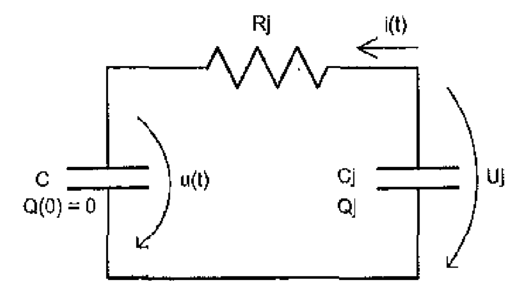

Figure 7. Schematic diagram showing the discharging of capacitor C_j into C through the resistor R_j.

By subtracting the asymptotic tangent from the $i(kT)$ graph, a new curve results, representing the current coming from the rest of the compartments, as shown in Figure 6. This curve is processed in the same way, considering that the last points represent the current coming from another single compartment. Thus, continuing this algorithm, one straight line corresponding to a single compartment results with each step. This iterative process ends when the curve resulting from the subtraction can be approximated with a straight line, meaning that it represents the current coming from the fastest compartment. Thus, the $i(kT)$ graph was decomposed into a sum of straight lines representing individual currents coming from virtual compartments. The number of compartments is equal to the number of straight lines. Because the currents I_{j0} start flowing out of compartments when the short is applied, the I_j terms represent the values of the currents flowing after the shorting period. Thus I_j can be replaced with

$$I_j = I_{j0} \exp\left[-\frac{t_{sc}}{\tau_j}\right] \tag{5}$$

where I_{j0} is the current flowing from compartment j at the moment the momentary short is applied, t_{sc} the shorting period, and τ_j is the time constant of compartment j. By replacing I_j in Equation (4), the recovery current is described by Equation (6)

$$i(t) = \sum_{j=1}^{m} I_{j0} \exp\left[-\frac{t_{sc}}{\tau_j}\right] \exp\left[-\frac{t}{\tau_j}\right] \tag{6}$$

The recovery voltage is calculated as

$$\begin{aligned} u(t) &= \frac{1}{C}\int i(t)\,dt \\ &= \frac{1}{C}\sum_{j=1}^{m} I_{j0} \exp\left[-\frac{t_{sc}}{\tau_j}\right] \tau_j \left(1 - \exp\left[-\frac{t}{\tau_j}\right]\right) \end{aligned} \tag{7}$$

The parameter I_{j0} is the initial value of the current corresponding to the virtual compartment j. Experimental results show that the I_{j0} value is proportional with the charging voltage while the τ_j time constant is fixed. Thus, the parameter I_{j0} can be expressed as the charging voltage U over a constant R_j

$$I_{j0} = \frac{U}{R_j} \tag{8}$$

The R_j constant is a resistance because it is defined as a voltage over a current. By replacing I_{j0} in Equation (7), the recovery voltage can be expressed as

$$u(t) = U_0 \frac{1}{C}\sum_{j=1}^{m} \left\{ \frac{\exp\left[-\frac{t_{sc}}{\tau_j}\right]}{R_j} \tau_j \left(1 - \exp\left[-\frac{t}{\tau_j}\right]\right) \right\} \tag{9}$$

where U is the charging voltage, C the capacitance, t_{sc} the momentary shorting period, τ_j the time constant and R_j is the resistance of compartment j. After the parameters R_j and τ_j are calculated for a particular charging voltage U, Equation (9) can be generalized to describe the recovery voltage for any charging voltage. Thus, after a capacitor is discharged from a voltage V_1 to V_2 where it is held for a period of time t_h, the voltage across its terminals rises following the equation

$$u(t) = \frac{|V_2 - V_1|}{C}\sum_{j=1}^{m} \left\{ \frac{\tau_j \exp\left[-\frac{t_{sc}}{\tau_j}\right]}{R_j} \left(1 - \exp\left[-\frac{t}{\tau_j}\right]\right) \right\} \tag{10}$$

where $|V_2 - V_1|$ is the absolute value of the voltage change on the capacitor. The hold time t_h is the equivalent of the momentary short. Similarly, after a capacitor is charged from a voltage V_1 to V_2 where it is held for a period of time t_h, the voltage across its terminals decreases following the equation

$$u(t) = \frac{|V_2 - V_1|}{C}\sum_{j=1}^{m} \left\{ \frac{\tau_j \exp\left[-\frac{t_{sc}}{\tau_j}\right]}{R_j} \left(\exp\left[-\frac{t}{\tau_j}\right] - 1\right) \right\} \tag{11}$$

Equations (10) and (11) represent the mathematical model of the voltage variation on a capacitor due to dielectric absorption. The parameters R_j and τ_j characterize each virtual compartment and m is the number of compartments. Based on the mathematical model, specific equations can be written for different charging and discharging patterns used in particular applications. The only constraint is the initial state of the capacitor which should be considered either fully charged or fully discharged.

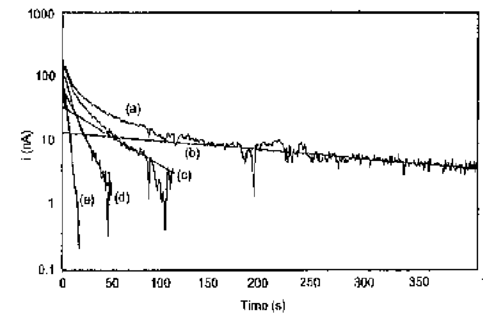

Figure 8. Semi-logarithmic plot showing the decomposition of the dielectric discharging current (a) in straight lines (b), (c), (d), (e), corresponding to exponential decay currents coming from four virtual compartments.

The equivalent circuit was built using the procedure described in Section (5), and is shown in Figure 9.

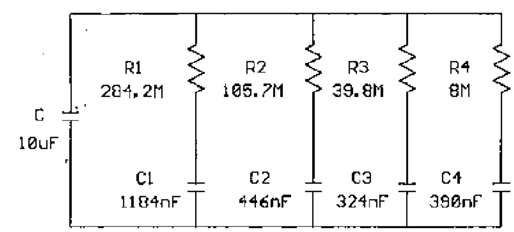

Figure 9. The equivalent electrical circuit of a 10 μF Z5U type ceramic capacitor.

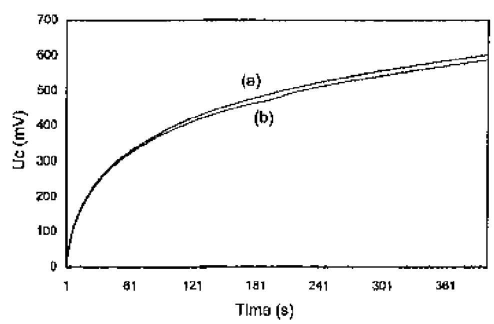

Figure 10. The mathematical model result (a) compared with the measured recovery voltage (b) during the data recording experiment.

5 THE EQUIVALENT ELECTRICAL CIRCUIT

The electrical circuit model is based on the similarity between the terms of Equation (4) and the discharging current of a capacitor through a resistor. To emphasize this similarity, consider a capacitance C_j which discharges through a resistor R_j into a capacitor C, as shown in Figure 7. The initial voltage on the capacitor C is 0 V and on C_j is U_j V. Current starts flowing through the resistor R_j charging the capacitor

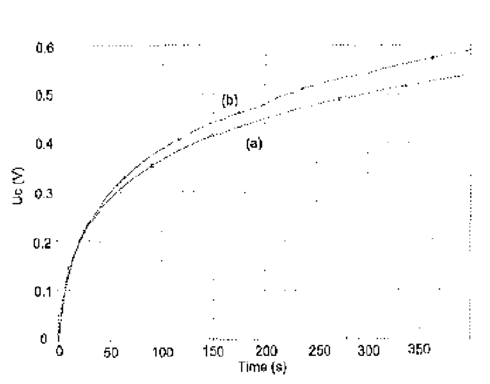

Figure 11. PSPICE simulation of the recovery voltage using the equivalent electrical circuit (a) compared with the measured recovery voltage (b) during the data recording experiment.

C with energy stored in C_j. This charging process ends when the voltage on capacitor C equals that on C_j. The voltage variation on C is described by the equation

$$u(t) = \frac{U_j C_j}{C_j + C}\left\{1 - \exp\left[-\frac{t}{R_j C_j C/(C_j + C)}\right]\right\} \quad (12)$$

Thus, the current $i(t)$ can be calculated as

$$i(t) = C\frac{du}{dt} = \frac{U_j}{R_j}\exp\left[-\frac{t}{R_j C_j C/(C_j + C)}\right] \quad (13)$$

Equation (13) has the same form as the terms in Equation (4). Thus, the circuit shown in Figure 7 can be used to model the discharging current from a single compartment, represented by R_j and C_j, into the conductive plates represented by C. From the equivalence between Equation (13) and the terms in Equation (4), R_j and C_j, are calculated as

$$R_j = \frac{U \exp\left[-\frac{t_{sc}}{\tau_j}\right]}{I_j} \quad (14)$$

$$C_j = \frac{C\tau_j}{R_j C - \tau_j} \quad (15)$$

The electrical circuit model contains the same number of compartments as the mathematical model. Each compartment consists of a series resistor-capacitor circuit, and all compartments are connected in parallel with the main capacitor. This equivalent circuit is similar to the one shown in Figure 1, where C is the capacitor value, m the number of compartments, and C_j and R_j, with $j = 1, 2, 3 \ldots m$, are the capacitance and resistance corresponding to each compartment.

There is a limitation of the equivalent circuit model. This limitation is caused by the current flowing between compartments. The equivalent circuit model is based on the mathematical model, and the mathematical model assumes that the current flows only from compartments into the conductive plates not between compartments. In the equivalent circuit model the first compartment that discharges will then recharge with current coming from the rest of the compartments. This recharge causes errors and limits the usage of the equivalent electric circuit to a timing less or equal to the discharging period of the fastest compartment. However, this period is much longer than the timing used in most analog applications. Thus, for these applications, the electrical model accurately represents the real capacitor, and it can be used to design compensation circuits.

6 ERROR ANALYSIS

The errors affecting the compartmental analysis of the dielectric absorption are due to the experimental data recording and the graphical curve fitting method. The data recording errors are caused by the input bias current into the voltmeter, the accuracy of the measured values, and the accuracy of the sampling period. The input bias current is subtracted from the recovery current during the experimental data recording. Thus, less current will recharge the conductive plates causing errors in the measured values. The accuracy of the voltmeter and the sampling period affects the measured values and the calculation of the recovery current. The curve fitting method adds errors due to the tangent drawing and the slope and intercept calculation. A good understanding of these errors help in setting up the experiment and building a more accurate model.

7 EXPERIMENTAL RESULTS

This Section presents a study done on a 10 μF Z5U type ceramic capacitor. The circuit presented in Figure 4 was used to charge the capacitor for 1 h and discharge it for 3 s. The voltage on the capacitor was measured using a voltmeter with 10 pA maximum input bias current and 0.1 mV accuracy. The data was recorded with a 1 s sampling rate for a period of 1 h. The recovery current was calculated using Equation (2) and was plotted on semi-logarithmic graph paper. The curve decomposition into straight lines followed the procedure described in Section 4 and the result is shown in Figure 8. It can be observed that the decomposition of the curve contains four straight lines. Only the first six minutes of recording period are shown to emphasize the section were the last three straight lines are located (lines c, d and e). The model consists of four compartments corresponding to each straight line. The discharging model was calculated following the procedure presented in Section 4 and is described by

$$\begin{aligned} u(t) = \Delta V[&106e^{-0.003t_h}(1 - e^{-0.003t}) \\ &+ 42e^{-0.022t_h}(1 - e^{-0.022t}) \\ &+ 31e^{-0.08t_h}(1 - e^{-0.08t}) \\ &+ 37e^{-0.333t_h}(1 - e^{-0.333t})] \end{aligned} \quad (16)$$

where $u(t)$ (in mV) is the voltage increase on the capacitor after a ΔV discharge and a t_h hold time of the final voltage value. The charging model is

$$\begin{aligned} u(t) = \Delta V[&106e^{-0.003t_h}(e^{-0.003t} - 1) \\ &+ 42e^{-0.022t_h}(e^{-0.022t} - 1) \\ &+ 31e^{-0.08t_h}(e^{-0.08t} - 1) \\ &+ 37e^{-0.333t_h}(e^{-0.333t} - 1)] \end{aligned} \quad (17)$$

The mathematical and electrical circuit models were evaluated using the experiment setup shown in Figure 4. The measured and calculated voltage variations on the capacitor are presented in Figure 10. The equivalent circuit was evaluated using PSPICE [5] simulation. The measured and simulated data are shown in Figure 11. It can be observed that the error increases after the first compartment is discharged and current starts to flow between compartments.

8 CONCLUSIONS

THIS paper presents a general method of studying the dielectric absorption in capacitors and building mathematical and electrical circuit models. The method is based on experimental data recording, and analysis combining mathematical, graphical and statistical techniques. The accuracy of the models depend on both experimental and data processing factors. The electrical circuit model has a timing limitation period related to the fastest time constant of compartments, after which the errors start to increase. However, the period of high accuracy is enough long compared to the timing used in most applications, and the electrical circuit model can be used with good results. The method presented in this paper can be used to study and model any type and value of capacitor, at slow or fast charging and discharging rates.

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