The reference and DUT signals are kept in quadrature. Assume the reference signal phase angle is $\frac{\pi}{2}$ larger than the DUT phase angle. Both oscillate with the same fundamental frequency ω_0, but the DUT signal phase angle includes an additional $\varphi(t)$ time dependent phase noise factor. Also, assume the mixer output is passed through a low-pass filter that eliminates frequencies significantly above ω_0, especially, $2 * \omega_0$.

$V_R(t) = V_{R-AMP} \sin\left(\omega_0 t + \frac{\pi}{2}\right)$, where V_{R-AMP} is the time-constant amplitude of the reference signal.

$V_{DUT}(t) = [V_{DUT-AMP} + \epsilon(t)] \sin(\omega_0 t + \varphi(t))$, where $V_{DUT-AMP}$ is the time-constant component of the DUT amplitude and $\epsilon(t)$ represents the time-dependent noise component.

Using K_φ to represent the conversion gain/loss of the mixer, its output is:

$$V_R(t)V_{DUT}(t) = K_\varphi V_{R-AMP} \sin\left(\omega_0 t + \frac{\pi}{2}\right) [V_{DUT-AMP} + \epsilon(t)] \sin(\omega_0 t + \varphi(t))$$

$$= \frac{K_\varphi V_{R-AMP} V_{DUT-AMP}}{2}\left(1 + \frac{\epsilon(t)}{V_{DUT-AMP}}\right)\left[\cos\left(\omega_0 t + \frac{\pi}{2} - \omega_0 t - \varphi(t)\right) - \cos\left(\omega_0 t + \frac{\pi}{2} + \omega_0 t + \varphi(t)\right)\right]$$

$$= \frac{K_\varphi V_{R-AMP} V_{DUT-AMP}}{2}\left(1 + \frac{\epsilon(t)}{V_{DUT-AMP}}\right)\left[\cos\left(\frac{\pi}{2} - \varphi(t)\right) - \cos\left(2\omega_0 t + \varphi(t) + \frac{\pi}{2}\right)\right]$$

Since $\cos\left(\frac{\pi}{2} - \varphi(t)\right) = \sin(\varphi(t))$ and $\cos\left(2\omega_0 t + \varphi(t) + \frac{\pi}{2}\right) = -\sin(2\omega_0 t + \varphi(t))$,

$$= \frac{K_\varphi V_{R-AMP} V_{DUT-AMP}}{2}\left(1 + \frac{\epsilon(t)}{V_{DUT-AMP}}\right)\left[\sin(\varphi(t)) + \sin(2\omega_0 t + \varphi(t))\right]$$

Assume $max(|\varphi(t)|) \ll 1\, radian$ (which is true when $\varphi(t)$ represents phase noise), then $\sin(\varphi(t)) \approx \varphi_{no\ units}(t)$. $\varphi(t)$ is expressed in radians, which are dimensionless. The notation $\varphi_{no\ units}$ makes this explicit.

Notice that $\sin(2\omega_0 t + \varphi(t))$ specifies a frequency that is twice the nominal frequency of the oscillator and reference, so it will be filtered out by the low pass filter. Identifying the signal from the low pass filter as $v(t)$,

$$v(t) \approx \frac{K_\varphi V_{R-AMP} V_{DUT-AMP}}{2}\left(1 + \frac{\epsilon(t)}{V_{DUT-AMP}}\right)\varphi_{no\ units}(t)$$

Assuming the amplitude of the DUT signal is relatively stable, that is $\frac{|\epsilon(t)|}{V_{DUT-AMP}} \ll 1$, we can write:
$v(t) \approx \frac{K_\varphi V_{R-AMP} V_{DUT-AMP}}{2} \varphi_{no\ units}(t)$ or, $v(t) \approx K_d\, \varphi_{no\ units}(t)$, where K_d is known as the phase discriminator constant, $K_d = \frac{K_\varphi V_{R-AMP} V_{DUT-AMP}}{2}$. (NB: conversion gain/loss, expressed as K_φ, is generally stated in terms of power gain/loss, so any value specified for a passive mixer would have to be converted to voltage units.)

If the reference signal follows the DUT signal in quadrature, the above math still works, except the resulting voltage will be negative. That is, the signal from the low pass filter would be: $v(t) \approx -K_d\, \varphi_{no\ units}(t)$.