Example 5.11. **Nonergodic Stationary Random Process.** A simple example of a nonergodic stationary random process follows. Consider a hypothetical random process $\{x_k(t)\}$ composed of sinusoidal sample functions such that

$$\{x_k(t)\} = \{X_k \sin[2\pi ft + \theta_k]\}$$

Let the amplitude X_k and the phase angle θ_k be random variables that take on a different set of values for each sample function, as illustrated in Figure 5.7. If θ_k is uniformly distributed, the properties of the process computed over the ensemble at specific times will be independent of time; hence the process is stationary. The properties computed by time averaging over individual sample functions are not always the same, however. For example, the autocovariance (or autocorrelation) function for each sample function is given here by

$$C_{xx}(\tau, k) = \frac{X_k^2}{2} \sin 2\pi f\tau$$

Because X_k is a function of k, $C_{xx}(\tau, k) \neq C_x(\tau)$. Hence, the random process is nonergodic.

Instead of having random amplitudes $\{X_k\}$, suppose each amplitude is the same X independent of k. Now the random process consists of sinusoidal sample functions such that

$$\{x_k(t)\} = \{X \sin(2\pi ft + \theta_k)\}$$

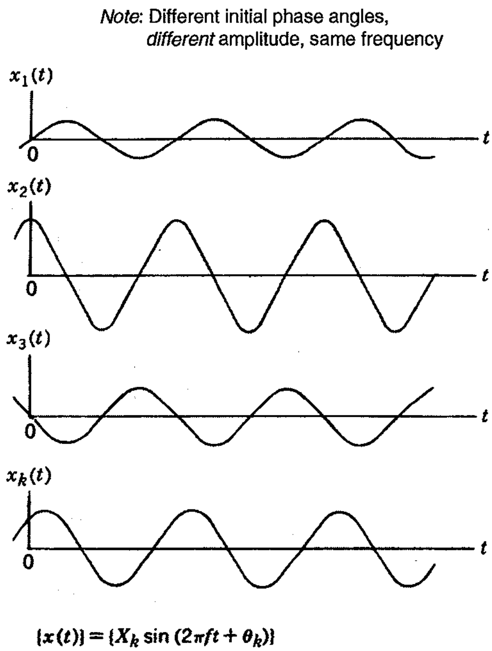

Figure 5.7 Illustration of nonergodic stationary sine wave process.

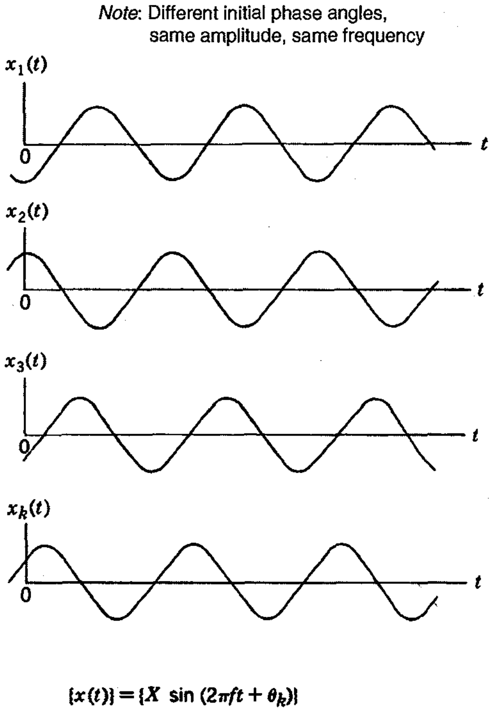

Figure 5.8 Illustration of ergodic sine wave process.

For this case, the random process is ergodic, with each record statistically equivalent to every other record for any time-averaging results, as illustrated in Figure 5.8. This concludes Example 5.11.

5.3.2 Sufficient Condition for Ergodicity

There are two important classes of random processes that one can state in advance will be ergodic. The first ergodic class is the class of stationary Gaussian random processes whose autospectral density functions are absolutely continuous; that is, no delta functions appear in the autospectra corresponding to infinite spectral densities at discrete frequencies. The second ergodic class (a special case of the first class) is the class of stationary Gaussian Markov processes; a Markov process is one whose relationship to the past does not extend beyond the immediately preceding observation. The autocorrelation function of a stationary Gaussian Markov process may be shown to be of a simple exponential form [Ref. 4].

Sufficient conditions for a random process to be ergodic are as follows:

I. A sufficient condition for an arbitrary random process to be weakly ergodic is that it be weakly stationary and that the time averages $\mu_x(k)$ and $C_{xx}(\tau, k)$ be the same for all sample functions k.