1.3 The Nonlinear Resistor

Recall that a resistor that is not linear is said to be nonlinear. A typical example of a nonlinear resistor is a germanium diode. For the *pn*-junction diode shown in Fig. 1.8, the branch current is a nonlinear function of the branch voltage, according to

$$i(t) = I_s(\epsilon^{qv(t)/kT} - 1) \tag{1.8}$$

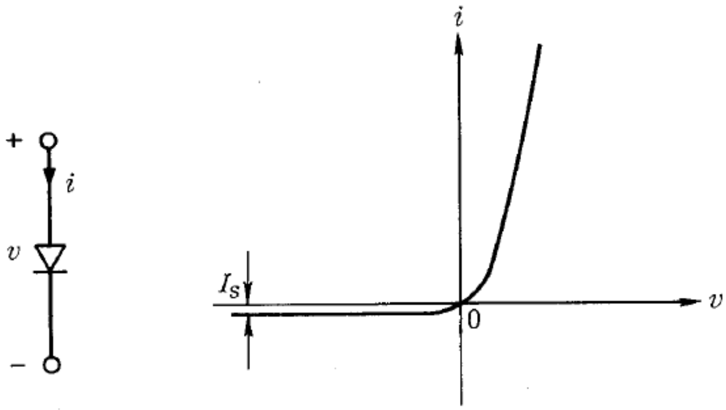

Fig. 1.8 Symbol for a *pn*-junction diode and its characteristic plotted in the *vi* plane.

where I_s is a constant that represents the reverse saturation current, i.e., the current in the diode when the diode is reverse-biased (i.e., with v negative) with a large voltage. The other parameters in (1.8) are q (the charge of an electron), k (Boltzmann's constant), and T (temperature in degrees Kelvin). At room temperature the value of kT/q is approximately 0.026 volt. The vi-plane characteristic is also shown in Fig. 1.8.

Exercise Plot the characteristic of a typical *pn*-junction diode in the vi plane by means of Eq. (1.8). Given $I_s = 10^{-4}$ amp, $kT/q \approx 0.026$ volt.

By virtue of its nonlinearity, a nonlinear resistor has a characteristic that is not at all times a straight line through the origin of the vi plane. Other typical examples of nonlinear two-terminal devices that may be modeled as nonlinear resistors are the tunnel diode and the gas tube. Their characteristics are shown in the vi plane of Figs. 1.9 and 1.10. Note

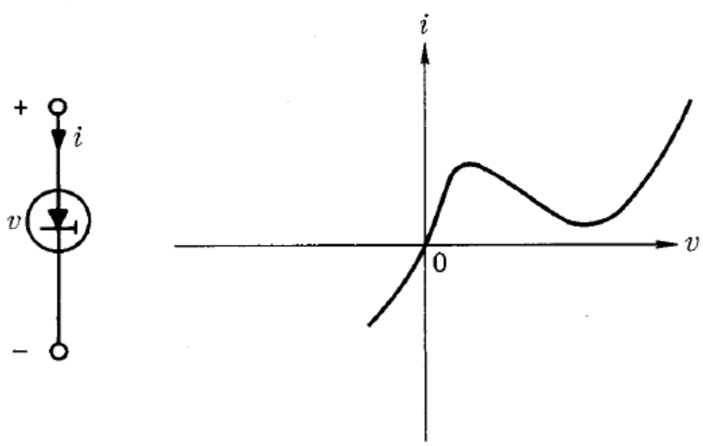

Fig. 1.9 Symbol for a tunnel diode and its characteristic plotted in the vi plane.

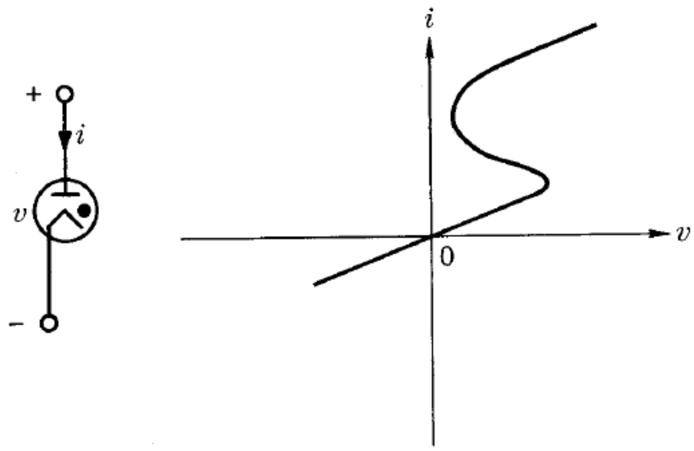

Fig. 1.10 Symbol for a gas diode and its characteristic plotted in the vi plane.

that in the first case the current i is a (single-valued) function of the voltage v; consequently, we can write $i = f(v)$. Indeed, as shown by the characteristic for each value of the voltage v, there is one and only one value possible for the current.† Such a resistor is said to be **voltage-controlled.** On the other hand, in the characteristic of the gas tube the voltage v is a (single-valued) function of the current i because for each i there is one and only one possible value of v. Thus, we can write $v = g(i)$. Such a resistor is said to be **current-controlled.** These nonlinear devices have a unique property in that the slope of the characteristic is negative in some range of voltage or current; they are often called negative-resistance devices and are of importance in electronic circuits. They can be used in amplifier circuits, oscillators, and computer circuits. The diode, the tunnel diode, and the gas tube are time-invariant resistors because their characteristics do not vary with time.

A nonlinear resistor can be both voltage-controlled and current-controlled as shown by the characteristic of Fig. 1.11. We can characterize such a resistor by either $i = f(v)$ or $v = g(i) = f^{-1}(i)$, where g is the function inverse to f. Note that the slope df/dv in Fig. 1.11 is positive for all v; we call such a characteristic *monotonically increasing.* A linear resistor with positive resistance is a special case of such a resistor; it has the monotonically increasing characteristic and is both voltage-controlled and current-controlled.

To analyze circuits with nonlinear resistors, we often depend upon the method of piecewise linear approximation. In this approximation nonlinear characteristics are approximated by piecewise straight-line segments. An often-used model in piecewise linear approximation is the **ideal diode.** A two-terminal nonlinear resistor is called an ideal diode if its

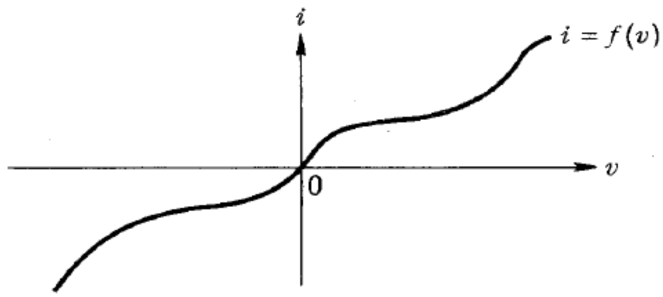

Fig. 1.11 A resistor which has a monotonically increasing characteristic is both voltage-controlled and current-controlled.

characteristic in the vi plane consists of two straight-line segments, the negative v axis and the positive i axis. The symbols of the ideal diode and its characteristic are shown in Fig. 1.12. When $v < 0$, $i = 0$; that is, for negative voltages the ideal diode behaves as an open circuit. When $i > 0$, $v = 0$; that is, for positive currents the ideal diode behaves as a short circuit.

At this point it is appropriate to introduce a distinct property of the linear resistor that is not usually present in the nonlinear resistor. A resistor is called **bilateral** if its characteristic is a curve that is symmetric with respect to the origin; in other words, whenever the point (v,i) is on the characteristic, so is the point $(-v,-i)$. Clearly, all linear resistors are bilateral, but most nonlinear resistors are not. It is important to realize the physical consequence of the bilateral property. For a bilateral element it is not important to keep track of the two terminals of the element; the element can be connected to the remainder of the circuit in either way. However, for a nonbilateral element such as a diode, one must know the terminal designation exactly.